CONTENTS

PRI	EFAC	CE	xvií
PA]	RT I	MODELING	1
1	Intr	oduction	3
	1.1	Integer Programming, 3	
	1.2	Standard Versus Nonstandard Forms, 5	
	1.3	Combinatorial Optimization Problems, 7	
	1.4	Successful Integer Programming Applications, 8	
	1.5	Text Organization and Chapter Preview, 8	
	1.6	Notes, 17	
	1.7	Exercises, 18	
2	Mod	deling and Models	21
	2.1	Assumptions on Mixed Integer Programs, 22	
	2.2	Modeling Process, 28	
	2.3	Project Selection Problems, 30	
		2.3.1 Knapsack Problem, 30	
		2.3.2 Capital Budgeting Problem, 31	
	2.4	Production Planning Problems, 32	
		2.4.1 Uncapacitated Lot Sizing, 33	
		2.4.2 Capacitated Lot Sizing, 34	
		2.4.3 Just-in-Time Production Planning, 34	

viii CONTENTS

2.5	2.5.1	orce/Staff Scheduling Problems, 36 Scheduling Full-Time Workers, 36			
2.6	2.6.1	Scheduling Full-Time and Part-Time Workers, 37 Charge Transportation and Distribution Problems, 38 Fixed-Charge Transportation, 38 Uncapacitated Facility Location, 40			
		Capacitated Facility Location, 41			
2.7		ommodity Network Flow Problem, 41			
2.8		rk Optimization Problems with Side Constraints, 43			
2.9		Chain Planning Problems, 44			
2.10	Notes,	47			
2.11	Exercis	ses, 48			
Tran	sformat	ion Using 0-1 Variables	54		
3.1	Transfe	orm Logical (Boolean) Expressions, 55			
	3.1.1	Truth Table of Boolean Operations, 55			
	3.1.2	Basic Logical (Boolean) Operations on Variables, 56			
	3.1.3	Multiple Boolean Operations on Variables, 58			
3.2	Transfe	orm Nonbinary to 0-1 Variable, 58			
	3.2.1	Transform Integer Variable, 58			
	3.2.2	Transform Discrete Variable, 60			
3.3	Transfe	orm Piecewise Linear Functions, 60			
	3.3.1	Arbitrary Piecewise Linear Functions, 60			
	3.3.2	Concave Piecewise Linear Cost Functions:			
		Economy of Scale, 63			
3.4	Transfe	orm 0–1 Polynomial Functions, 64			
3.5	Transform Functions with Products of Binary and Continuous				
		es: Bundle Pricing Problem, 66			
3.6	Transfe	orm Nonsimultaneous Constraints, 69			
	3.6.1	Either/Or Constraints, 69			
	3.6.2	p Out of m Constraints Must Hold, 70			
		Disjunctive Constraint Sets, 71			
		Negation of a Constraint, 71			
		If/Then Constraints, 71			
3.7					
3.8	Exercis	ses, 73			
Bette	r Form	ulation by Preprocessing	79		
4.1	Better	Formulation, 79			
4.2	Autom	atic Problem Preprocessing, 86			
4.3	Tightening Bounds on Variables, 87				
	4.3.1	Bounds on Continuous Variables, 87			
	4.3.2	Bounds on General Integer Variables, 88			
	4.3.3	Bounds on 0-1 Variables, 90			

CONTENTS ix

	4.3.4	Variable Fixing, Redundant Constraints, and Infeasibility, 91	
4.4	Prepro	cessing Pure 0–1 Integer Programs, 93	
	4.4.1	Fixing 0-1 Variables, 93	
	4.4.2	Detecting Redundant Constraints And Infeasibility, 95	
		Tightening Constraints (or Coefficients Reduction), 96	
	4.4.4		
	4.4.5	Rounding by Division with GCD, 98	
4.5	Decon	posing a Problem into Independent Subproblems, 99	
4.6		g the Coefficient Matrix, 100	
4.7	Notes,		
4.8	Exerci	ses, 101	
Mod	leling C	Combinatorial Optimization Problems I	105
5.1	Introdi	action, 105	
5.2		overing and Set Partitioning, 106	
	5.2.1	•	
		Set Partitioning and Set Packing, 111	
		Set Covering in Networks, 111	
		Applications of Set Covering Problem, 113	
5.3		ing Problem, 115	
		Matching Problems in Network, 115	
	5.3,2	Integer Programming Formulation, 116	
5.4	Cuttin	g Stock Problem, 117	
	5.4.1	One-Dimensional Case, 117	
	5.4.2	Two-Dimensional Case, 120	
5.5	Compa	arisons for Above Problems, 121	
5.6	Comp	utational Complexity of COP, 121	
	5.6.1	Problem Versus Problem Instance, 123	
	5.6.2	Computational Complexity of an Algorithm, 123	
		Polynomial Versus Nonpolynomial Function, 124	
5.7	Notes,		
5.8	Exerci	ses, 126	
Mod	deling (Combinatorial Optimization Problems II	130
6.1	Impor	tance of Traveling Salesman Problem, 130	
6.2		formations to Traveling Salesman Problem, 133	
J. _	6.2.1	Shortest Hamiltonian Paths, 133	
	6.2.2	TSP with Repeated City Visits, 134	
	6.2.3	Multiple Traveling Salesmen Problem, 135	
	6.2.4		
	6.2.5	Generalized TSP, 137	
	6.2.6		

CONTENTS X

	6.3		
		6.3.1 Machine Sequencing Problems in Various	
		Manufacturing Systems, 140	
		6.3.2 Sequencing Problems in Electronic Industry, 140	
		6.3.3 Vehicle Routing for Delivery/Dispatching, 141	
		6.3.4 Genome Sequencing for Genetic Study, 142	
	6.4	,	
		6.4.1 Subtour Elimination by Dantzig-Fulkerson-	
		Johnson Constraints, 143	
		6.4.2 Subtour Elimination by Miller-Tucker-Zemlin	
	2.3	(MTZ) Constraints, 144	
		Formulating Symmetric TSP, 146	
		Notes, 148	
	6.7	Exercises, 149	
PAI	RT II	REVIEW OF LINEAR PROGRAMMING	
AN	D NE	TWORK FLOWS	153
7	Line	ear Programming—Fundamentals	155
	7.1	Daview of Bosic Lingue Algebra 155	
	1.1	Review of Basic Linear Algebra, 155 7.1.1 Euclidean Space, 155	
		7.1.1 Euchdean Space, 133 7.1.2 Linear and Convex Combinations, 156	
		7.1.3 Linear Independence, 156	
		7.1.4 Rank of a Matrix, 156	
		7.1.5 Basis, 157	
		7.1.6 Matrix Inversion, 157	
		7.1.7 Determinant of a Matrix, 157	
		7.1.8 Upper and Lower Triangular Matrices, 158	
	7.2		
	, . 2	7.2.1 Finding the Rank of a Matrix, 159	
		7.2.2 Calculating the Inverse of a Matrix, 160	
		7.2.3 Converting to a Triangular Matrix, 161	
		7.2.4 Calculating the Determinant of a Matrix, 162	
		7.2.5 Solving a System of Linear Equations, 162	
	7.3	The Dual Linear Program, 165	
		7.3.1 The Linear Program in Standard Form, 166	
		7.3.2 Formulating the Dual Problem, 167	
		7.3.3 Economic Interpretation of the Dual, 170	
		7.3.4 Importance of the Dual, 171	
	7.4	Relationships Between Primal and Dual Solutions, 171	
		7.4.1 Relationships Between All Primal and All	
		Dual Feasible Solutions, 171	
		7.4.2 Relationship Between Primal and Dual	
		Optimum Solutions, 172	

CONTENTS xi

		7.4.3	Relationships Between Each Complementary Pair	
	7.5	Notes,	of Variables at Optimum, 173	
	7.6		ses, 176	
	7.0	EXCICI	565, 170	
8	Line	ar Pro	gramming: Geometric Concepts	180
	8.1	Geome	etric Solution, 180	
		8.1.1	Objective Function, 181	
		8.1.2	Solution Space, 181	
		8.1.3	Requirement Space, 183	
	8.2	Conve	x Sets, 188	
			Convex Sets and Polyhedra, 188	
			Directions of Unbounded Convex Sets, 191	
			Convex and Polyhedral Cones, 191	
			Convex and Concave Functions, 192	
	8.3	Descri	bing a Bounded Polyhedron, 194	
		8.3.1	,	
		8.3.2	· · · · · · · · · · · · · · · · · · ·	
	8.4		bing Unbounded Polyhedron, 195	
		8.4.1	,,,,,	
			Representing by Extreme Points and Extreme Directions,	199
			Example of Representation Theorem, 199	
	8.5		Facets, and Dimension of a Polyhedron, 199	
	8.6		bing a Polyhedron by Facets, 201	
	8.7		spondence Between Algebraic and Geometric Terms, 202	
	8.8	Notes,		
	8.9	Exerci	ises, 203	
9	Line	ear Pro	gramming: Solution Methods	207
	9.1	Linear	Programs in Canonical Form, 207	
	9.2	Basic	Feasible Solutions and Reduced Costs, 209	
		9.2.1	Basic Feasible Solution, 209	
		9.2.2	Adjacent Basic Feasible Solution, 211	
		9.2.3	Reduced Costs, 212	
	9.3		implex Method, 213	
		9.3.1	Better and Feasible Solution, 213	
		9.3.2	Updating Simplex Tableau by Pivoting, 215	
		9.3.3	Optimality Test, 216	
		9.3.4	Initial Basic Feasible Solution, 216	
	9.4	-	reting the Simplex Tableau, 218	
		9.4.1	Entire Simplex Tableau, 218	
		9.4.2	Rows of Simplex Tableau, 218	
		9.4.3	Columns of Simplex Tableau, 219	
		9.4.4	Pivot Column and Pivot Row, 219	

xii CONTENTS

	9.5 9.6 9.7 9.8 9.9 9.10	9.4.5 Predicting the New Objective Value Before Updating, 219 Geometric Interpretation of the Simplex Method, 220 9.5.1 Basic Feasible Solution Versus Extreme Point, 220 9.5.2 Explanation of "Simplex Method" Nomenclature, 222 9.5.3 Identifying an Extreme Ray in a Simplex Tableau, 223 The Simplex Method for Upper Bounded Variables, 227 The Dual Simplex Method, 231 The Revised Simplex Method, 233 Notes, 239 Exercises, 240	
10	Netwo	ork Optimization Problems and Solutions	246
	10.1	Network Fundamentals, 247	
	10.2	A Class of Easy Network Problems, 248	
		10.2.1 The Minimum Cost Network Flow Problem, 249	
		10.2.2 Formulating the Transportation-Assignment Problem	
		as an MCNF Problem, 249	
		10.2.3 Formulating the Transshipment Problem	
		as an MCNF Problem, 251	
		10.2.4 Formulating the Maximum Flow Problem	
		as an MCNF Problem, 251	
		10.2.5 Formulating the Shortest Path Problem	
		as an MCNF Problem, 251	
	10.3	Totally Unimodular Matrices, 252	
		10.3.1 Definition, 252	
		10.3.2 Sufficient Condition for a Totally Unimodular Matrix, 252	
		10.3.3 Some Properties of Totally Unimodular Matrices, 254	
		10.3.4 Matrix Structure of the MCNF Problem, 254	
		10.3.5 Lower Triangular Matrix and Forward Substitution, 255	
		10.3.6 Naturally Integer Solution for the MCNF Problem, 255	
	10.4	The Network Simplex Method, 256	0.000
		10.4.1 Feasible Spanning Trees Versus Basic Feasible Solutions,	256
		10.4.2 The Network Algorithm, 257	
		10.4.3 Numerical Example, 258	
	10.5	Solution via LINGO, 264	
	10.6	Notes, 264	
	10.7	Exercises, 265	
PAI	RT III	SOLUTIONS	269
11	Classi	ical Solution Approaches	271
	11.1	Branch-and-Bound Approach, 272	
		11.1.1 Basic Concepts, 272	
		11.1.2 Branch-and-Bound Algorithm, 278	

CONTENTS xiii

11,2	Cutting	Plane Approach, 280	
	11.2.1	Dual Cutting Plane Approach, 280	
	11.2.2	Fractional Cutting Plane Method, 281	
	11.2.3	Mixed Integer Cutting Plane Method, 285	
11.3		Theoretic Approach, 286	
	11.3.1	Group Theory Terminology, 287	
	11.3.2		
	11.3.3	Formulating a Group Problem, 290	
	11.3.4		
		Route Problem, 291	
	11.3.5		
11.4		tric Concepts, 294	
		Various Polyhedrons in Original Space, 295	
	11.4.2		
		of Nonbasic Variables, 297	
11.5	Notes,		
11.6	Exercis		
Bran	ch-and-0	Cut Approach	305
12.1	Introdu	ction, 306	
12.1		Basic Concept, 306	
		Branch-and-Cut Algorithm, 306	
		Generating Valid Cuts and Preprocessing, 307	
12.2		nequalities, 308	
12.2		Valid Inequalities for Linear Programs, 308	
		Valid Inequalities for Integer Programs, 308	
	12.2.3		
12.3		nerating Techniques, 309	
12.5	12.3.1	Rounding Technique, 310	
		Disjunction Technique, 310	
		Lifting Technique, 312	
12.4		enerated from Sets Involving Pure Integer Variables, 313	
		Gomory Fractional Cut, 313	
		Chvátal-Gomory Cut, 313	
		Pure Integer Rounding Cut, 314	
		Objective Integrality Cut, 315	
12.5		enerated from Sets Involving Mixed Integer Variables, 315	i
	12.5.1	Gomory Mixed Integer Cut, 315	
	12.5.2	Mixed Integer Rounding Cut, 319	
12.6	Cuts G	enerated from 0-1 Knapsack Sets, 320	
	12.6.1	Knapsack Cover, 320	
		Lifted Knapsack Cover, 321	
		GUB Cover, 323	
12.7	Cuts G	enerated from Sets Containing 0-1 Coefficients	
		I Variables, 324	

xiv CONTENTS

	12.8	Cuts Generated from Sets with Special Structures, 326 12.8.1 Flow Cover from Fixed-Charge Flow Network, 326	
		12.8.2 Plant/Facility Location (Fixed-Charge Transportation),	327
	12.9	Notes, 329	321
		Exercises, 330	
	12.10	Entition, 500	
13	Branc	h-and-Price Approach	334
	13.1	Concepts of Branch-and-Price, 334	
	13.2	Dantzig-Wolfe Decomposition, 335	
	13.3	Generalized Assignment Problem, 344	
		13.3.1 Conventional Formulation, 345	
		13.3.2 Column Generation Formulation, 345	
		13.3.3 Initial Solution, 348	
	13.4	GAP Example, 348	
		13.4.1 GAP Branching Scheme, 353	
		13.4.2 Tailing-Off Effect of Column Generation, 353	
		13.4.3 Treatment of Identical Machines, 354	
		13.4.4 Branch-and-Price Algorithm, 356	
	13.5	Other Application Areas, 356	
	13.6	Notes, 357	
	13.7	Exercises, 358	
	0.1.41		
14		on via Heuristics, Relaxations, and Partitioning	359
	14.1	Introduction, 359	
	14.2	Overall Solution Strategy, 359	
		14.2.1 Better Formulation by Preprocessing, 360	
		14.2.2 LP-Based Branch-and-Bound Framework, 361	
		14.2.3 Heuristics for Tightening Lower Bounds, 361	
		14.2.4 Relaxations for Tightening Upper Bounds, 362	
		14.2.5 Strong Cuts for Tightening Solution Polyhedron, 362	
	14.3	Primal Solution via Heuristics, 363	
		14.3.1 Local Search Approaches, 364	
	20000	14.3.2 Artificial Intelligence Approaches, 366	
	14.4	Dual Solution via Relaxation, 373	
		14.4.1 Linear Programming Relaxation, 373	
		14.4.2 Combinatorial Relaxation, 374	
		14.4.3 Lagrangian Relaxation, 376	
	14.5	Lagrangian Dual, 377	
		14.5.1 Lagrangian Dual in LP, 378	
		14.5.2 Lagrangian Dual in IP, 378	
		14.5.3 Properties of the Lagrangian Dual, 379	
	14.6	Primal-Dual Solution via Benders' Partitioning, 380	
	14.7	Notes, 383	
	14.8	Exercises, 383	

CONTENTS xv

15	Solut	ions with Commercial Software	386
	15.1	Introduction, 387	
	15.2	Typical IP Software Components, 388	
		15.2.1 Solvers, 388	
		15.2.2 Presolvers, 389	
		15.2.3 Modeling Languages, 389	
		15.2.4 User's Options/Intervention, 390	
		15.2.5 Data and Application Interfaces, 391	
	15.3	The AMPL Modeling Language, 392	
		15.3.1 Components of the AMPL Modeling Language, 392	
		15.3.2 An AMPL Example: the Diet Problem, 393	
		15.3.3 Enhanced AMPL Modeling Techniques, 397	
		15.3.4 AMPL Compatible MIP Solvers, 400	
	15.4	LINGO Modeling Language, 400	
		15.4.1 Prescription of Tolerances, 401	
		15.4.2 Presolver—Automatic Problem Reduction, 402	
		15.4.3 Solvers for Linear/Integer Programming, 402	
		15.4.4 Interfacing with the User, 403	
		15.4.5 LINGO Modeling Conventions, 403	
		15.4.6 LINGO Model for the Diet Problem, 404	
	15.5	MPL Modeling Language, 405	
		15.5.1 MPL Modeling Conventions, 406	
		15.5.2 MPL Model for the Diet Problem, 408	
		15.5.3 MPL Compatible MIP Solvers, 409	
RE	FERE	NCES	411
AP	APPENDIX: ANSWERS TO SELECTED EXERCISES		
INI	INDEX		