

Contents

Preface	xi
Acknowledgments	xiv
Notation	xv
I Basic Concepts	1
1 Introduction	3
1.1 The structured nonconvex mixed integer nonlinear program	3
1.2 Applications	4
1.3 Outline of the solution approach	5
1.4 An illustrative example	6
2 Problem Formulations	9
2.1 The condensed formulation	9
2.2 Smooth and disjunctive reformulations	10
2.2.1 Integrality constraints	10
2.2.2 Disjunctive constraints	10
2.2.3 Big-M constraints	11
2.2.4 The smooth binary formulation	11
2.2.5 Block-separability	12
2.3 Block-separable splitting-schemes	12
2.3.1 The sparsity graph	12
2.3.2 MINLP splitting-schemes	12
2.3.3 MIQQP splitting-schemes	14
2.4 Separable reformulation of factorable programs	15
2.5 Extended block-separable reformulation	17
2.6 Other formulations	18

3	Convex and Lagrangian Relaxations	21
3.1	Convexification of sets and functions	21
3.2	Convex underestimating-relaxations	23
3.3	Lagrangian relaxation	24
3.4	Dual-equivalent convex relaxations	25
3.5	Reducing the duality gap	28
3.6	Augmented Lagrangians	31
4	Decomposition Methods	33
4.1	Lagrangian decomposition — dual methods	33
4.1.1	Subgradient methods	35
4.1.2	Dual cutting-plane methods	36
4.1.3	Proximal bundle methods	38
4.2	Primal cutting-plane methods	39
4.3	Column generation	42
4.3.1	A simple column generation method	42
4.3.2	Initializing the RMP	45
4.3.3	An improved column generation method	49
4.4	Benders decomposition	52
5	Semidefinite Relaxations	55
5.1	Semidefinite and Lagrangian relaxations	55
5.2	Block-separable reformulation	58
5.3	Eigenvalue representation of the dual function	59
5.4	Duality results and convex relaxation	60
5.4.1	The trust region problem	60
5.4.2	Dual-equivalence	61
5.4.3	Modifications	63
5.4.4	Influence of decomposition on the dual function	64
5.5	Solving the Lagrangian dual problem (\tilde{D})	65
5.6	Numerical results	66
5.6.1	Block structure	66
5.6.2	Network structure	67
5.7	Computing relaxations of mixed linear quadratic programs	69
6	Convex Underestimators	73
6.1	Interval arithmetic	73
6.2	Bézier polynomials	75
6.3	α -underestimators	77
6.4	CGU-underestimators	78
6.5	Convexified polynomial underestimators	78
6.5.1	Rigorous underestimators	80
6.5.2	Restricted sampling	80

7	Cuts, Lower Bounds and Box Reduction	83
7.1	Valid cuts	83
7.1.1	Linearization cuts	84
7.1.2	Knapsack cuts	84
7.1.3	Interval-gradient cuts	85
7.1.4	Lagrangian cuts	86
7.1.5	Level cuts	87
7.1.6	Other valid cuts	87
7.2	Initialization of polyhedral relaxations	88
7.3	Lower bounds	88
7.3.1	NLP-bounds	89
7.3.2	MINLP-bounds	90
7.3.3	Dual bounds	90
7.3.4	LP-bounds	90
7.4	Box reduction	91
7.5	Numerical results	92
8	Local and Global Optimality Criteria	99
8.1	Local optimality conditions	99
8.2	Local strong duality of nonconvex QQPs	101
8.3	Global optimality cuts	105
8.4	Some global optimality criteria for QQPs	106
8.5	Global optimality via interval-gradient cuts	110
9	Adaptive Discretization of Infinite Dimensional MINLPs	113
9.1	Aggregated discretizations	113
9.1.1	Multistage stochastic programs	113
9.1.2	Optimal control problems	115
9.1.3	Abstract formulation	116
9.2	Optimal mesh and scenario refinement	116
9.3	Updating and solving relaxations	117
II	Algorithms	119
10	Overview of Global Optimization Methods	121
10.1	Sampling heuristics	123
10.2	Branch-and-bound methods	125
10.3	Successive approximation methods	126
10.4	Relaxation-based heuristics	127

11 Deformation Heuristics	129
11.1 The algorithm of Moré and Wu	129
11.2 A MaxCut deformation heuristic	130
11.2.1 Problem formulation	130
11.2.2 A MaxCut algorithm	132
11.2.3 Sampling	134
11.2.4 Numerical results	135
11.3 Generalization to MINLP	138
11.3.1 Parametric problem formulation	138
11.3.2 A MINLP deformation algorithm	139
11.3.3 Numerical results	140
12 Rounding, Partitioning and Lagrangian Heuristics	143
12.1 A rounding heuristic	143
12.2 A partitioning heuristic that uses central cuts	145
12.3 Numerical results	147
12.4 A Lagrangian heuristic	153
13 Branch-Cut-and-Price Algorithms	155
13.1 Branch-and-bound algorithms	155
13.1.1 Preliminaries	155
13.1.2 A generic branch-and-bound algorithm	156
13.2 Convergence and finiteness	156
13.2.1 Convergence	156
13.2.2 Finiteness	157
13.3 Consistent bounding operations	159
13.3.1 NLP-bounds	159
13.3.2 LP-bounds	160
13.3.3 Dual bounds	161
13.4 Branching	162
13.4.1 Rectangular subdivision rules	162
13.4.2 Updating lower bounds	163
13.5 Numerical results	163
13.5.1 Network MaxCut experiments	164
13.5.2 MINLP experiments	169
13.5.3 Cost-efficient design of energy conversion systems	175
13.6 Nonconvex polyhedral inner and outer approximations	176
14 LaGO — An Object-Oriented Library for Solving MINLPs	181
14.1 Design philosophy	181
14.2 Related work	182
14.3 Structure	183

14.4 The modules	183
14.4.1 Reformulation	183
14.4.2 Relaxation	185
14.4.3 Solvers	186
Appendix	189
A Future Perspectives	189
B MINLP Problems	191
B.1 Instances from the MINLPLIB	191
B.2 Random MIQQP problems	193
Bibliography	195
Index	211