

Contents

LIST OF FIGURES	xv
LIST OF TABLES	xvii
PREFACE	xix
DEFINITION OF SYMBOLS	xxiii
1 GEOMETRY OF LINEAR INEQUALITY SYSTEMS AND THE SIMPLEX METHOD	1
1.1 CONVEXITY AND LINEAR INEQUALITY SYSTEMS	1
1.1.1 Affine & Convex Combinations	1
1.1.2 Two-dimensional Convex Regions	3
1.1.3 Line Segments, Rays, and Half Lines	5
1.1.4 General Convex Regions	6
1.1.5 Hyperplanes and Half-Spaces	7
1.1.6 Convexity of Half Spaces and Hyperplanes	8
1.1.7 Convexity of the Set of Feasible Solutions of an LP	9
1.1.8 Convex Polyhedrons, Polytopes, and Cones	9
1.1.9 Separating Hyperplane	11
1.2 SIMPLEX DEFINED	13
1.3 GLOBAL MINIMUM, EXTREME POINTS, AND EDGES	14
1.4 THE SIMPLEX METHOD VIEWED AS THE STEEPEST DE- SCENT ALONG EDGES	20
1.5 THE SIMPLEX INTERPRETATION OF THE SIMPLEX METHOD	24
1.6 NOTES & SELECTED BIBLIOGRAPHY	31
1.7 PROBLEMS	31
2 DUALITY AND THEOREMS OF THE ALTERNATIVES	43
2.1 THE DUALITY THEOREM	43
2.2 ADDITIONAL THEOREMS ON DUALITY	47
2.2.1 Unboundedness Theorem	47
2.2.2 Miscellaneous Theorems for the Standard Form	48

2.3	COMPLEMENTARY SLACKNESS	49
2.4	THEOREMS OF THE ALTERNATIVES	50
2.4.1	Gordan's Theorem	51
2.4.2	Farkas's Lemma	52
2.4.3	Stiemke's Theorem	53
2.4.4	Motzkin's Transposition Theorem	54
2.4.5	Ville's Theorem	55
2.4.6	Tucker's Strict Complementary Slackness Theorem	56
2.5	NOTES & SELECTED BIBLIOGRAPHY	58
2.6	PROBLEMS	59
3	EARLY INTERIOR-POINT METHODS	67
3.1	VON NEUMANN'S METHOD	70
3.1.1	The von Neumann Algorithm	73
3.1.2	Improving the Rate of Convergence	81
3.1.3	Von Neumann Algorithm as a Variant of the Simplex Algorithm	83
3.2	DIKIN'S METHOD	84
3.2.1	Dikin's Algorithm	87
3.2.2	Convergence of Dikin's Algorithm	89
3.3	KARMAKAR'S METHOD	100
3.3.1	Development of the Algorithm	100
3.3.2	Proof of Convergence	105
3.3.3	The Algorithm Summarized	114
3.3.4	Converting a Standard LP to a Starting Form for the Algorithm	115
3.3.5	Computational Comments	116
3.3.6	Complexity of von Neumann versus Karmarkar Algorithms	118
3.4	NOTES & SELECTED BIBLIOGRAPHY	119
3.5	PROBLEMS	121
4	INTERIOR-POINT METHODS	123
4.1	NEWTON'S METHOD	123
4.2	THE LINEAR LEAST-SQUARES PROBLEM	127
4.3	BARRIER FUNCTION METHODS	128
4.3.1	The Logarithmic Barrier Function	128
4.3.2	Properties of Barrier Function Methods	130
4.4	THE PRIMAL LOGARITHMIC BARRIER METHOD FOR SOLVING LINEAR PROGRAMS	131
4.4.1	Details of the Method	131
4.4.2	Initial Feasible Solution	134
4.5	PRIMAL-DUAL LOGARITHMIC BARRIER METHODS	134
4.6	RECOVERING A BASIC FEASIBLE SOLUTION	137
4.7	COMPUTATIONAL COMMENTS	139
4.8	NOTES & SELECTED BIBLIOGRAPHY	140
4.9	PROBLEMS	146

5	DEGENERACY	149
5.1	EXAMPLES OF CYCLING	149
5.2	ON RESOLVING DEGENERACY	153
5.3	DANTZIG'S INDUCTIVE METHOD	154
5.4	WOLFE'S RULE	156
5.5	BLAND'S RULE	158
5.6	KRISHNA'S EXTRA COLUMN RULE	160
5.7	ON AVOIDING DEGENERATE PIVOTS	164
5.8	NOTES & SELECTED BIBLIOGRAPHY	166
5.9	PROBLEMS	167
6	VARIANTS OF THE SIMPLEX METHOD	173
6.1	INTRODUCTION	173
6.2	MAX IMPROVEMENT PER ITERATION	176
6.3	DUAL-SIMPLEX METHOD	179
6.4	PARAMETRIC LINEAR PROGRAMS	183
6.4.1	Parameterizing the Objective Function	183
6.4.2	Parameterizing the Right-Hand Side	187
6.5	SELF-DUAL PARAMETRIC ALGORITHM	188
6.6	THE PRIMAL-DUAL ALGORITHM	191
6.7	THE PHASE I LEAST-SQUARES ALGORITHM	197
6.8	NOTES & SELECTED BIBLIOGRAPHY	200
6.9	PROBLEMS	202
7	TRANSPORTATION PROBLEM AND VARIATIONS	207
7.1	THE CLASSICAL TRANSPORTATION PROBLEM	207
7.1.1	Mathematical Statement	208
7.1.2	Properties of the System	208
7.2	FINDING AN INITIAL SOLUTION	213
7.3	FINDING AN IMPROVED BASIC SOLUTION	214
7.4	DEGENERACY IN THE TRANSPORTATION PROBLEM	216
7.5	TRANSSHIPMENT PROBLEM	219
7.5.1	Formulation	219
7.5.2	Reduction to the Classical Case by Computing Minimum Cost Routes	222
7.5.3	Reduction to the Classical Case by the Transshipment Pro- cedure	222
7.6	TRANSPORTATION PROBLEMS WITH BOUNDED PARTIAL SUMS	225
7.7	NOTES & SELECTED BIBLIOGRAPHY	227
7.8	PROBLEMS	228

8	NETWORK FLOW THEORY	231
8.1	THE MAXIMAL FLOW PROBLEM	232
8.1.1	Decomposition of Flows	233
8.1.2	The Augmenting-Path Algorithm for Maximal Flow	234
8.1.3	Cuts in a Network	239
8.2	SHORTEST ROUTE	241
8.3	MINIMUM COST-FLOW PROBLEM	242
8.4	NOTES & SELECTED BIBLIOGRAPHY	243
8.5	PROBLEMS	245
9	GENERALIZED UPPER BOUNDS	251
9.1	PROBLEM STATEMENT	251
9.2	BASIC THEORY	253
9.3	SOLVING SYSTEMS WITH GUB EQUATIONS	253
9.4	UPDATING THE BASIS AND WORKING BASIS	257
9.5	NOTES & SELECTED BIBLIOGRAPHY	264
9.6	PROBLEMS	264
10	DECOMPOSITION OF LARGE-SCALE SYSTEMS	265
10.1	WOLFE'S GENERALIZED LINEAR PROGRAM	267
10.2	DANTZIG-WOLFE (D-W) DECOMPOSITION PRINCIPLE	280
10.2.1	D-W Principle	284
10.2.2	D-W Decomposition Algorithm and Variants	289
10.2.2.1	The D-W Algorithm	289
10.2.2.2	Variants of the D-W Algorithm	290
10.2.3	Optimality and Dual Prices	290
10.2.4	D-W Initial Solution	291
10.2.5	D-W Algorithm Illustrated	292
10.3	BENDERS DECOMPOSITION	299
10.3.1	Dual of D-W Decomposition	299
10.3.2	Derivation of Benders Decomposition	300
10.4	BLOCK-ANGULAR SYSTEM	306
10.5	STAIRCASE STRUCTURED PROBLEMS	308
10.5.1	Using Benders Decomposition	309
10.5.2	Using D-W Decomposition	310
10.5.3	Using D-W Decomposition with Alternate Stages Forming the Subproblems	312
10.6	DECOMPOSITION USED IN CENTRAL PLANNING	313
10.7	NOTES & SELECTED BIBLIOGRAPHY	315
10.8	PROBLEMS	317

11 STOCHASTIC PROGRAMMING: INTRODUCTION	323
11.1 OVERVIEW	324
11.2 UNCERTAIN COSTS	326
11.2.1 Minimum Expected Costs	326
11.2.2 Minimum Variance	327
11.3 UNCERTAIN DEMANDS	329
11.4 NOTES & SELECTED BIBLIOGRAPHY	332
11.5 PROBLEMS	332
12 TWO-STAGE STOCHASTIC PROGRAMS	335
12.1 THE DETERMINISTIC TWO-STAGE LP PROBLEM	335
12.2 THE ANALOGOUS STOCHASTIC TWO-STAGE LP PROBLEM	336
12.3 LP EQUIVALENT OF THE STOCHASTIC PROBLEM (EQ-LP)	337
12.3.1 LP Equivalent Formulation	337
12.3.2 Geometric Description of Benders Decomposition Algorithm	338
12.3.3 Decomposition Algorithm	341
12.3.4 Theory behind the Algorithm	348
12.4 SOLVING STOCHASTIC TWO-STAGE PROBLEMS USING SAM- PLING	349
12.4.1 Overview	349
12.4.2 Naive Sampling	350
12.4.3 Sampling Methodology	351
12.4.4 Estimating Upper Bound z_{UB} for Min z given $x = x^k$	351
12.4.5 Estimating Lower Bound z_{LB} for Min z	352
12.5 USE OF IMPORTANCE SAMPLING	354
12.5.1 Crude (Naive) Monte Carlo Methods	355
12.5.2 Monte Carlo Methods using Importance Sampling	356
12.6 NOTES & SELECTED BIBLIOGRAPHY	360
12.7 PROBLEMS	362
A PROBABILITY THEORY: OVERVIEW	367
A.1 BASIC CONCEPTS, EXPECTED VALUE, AND VARIANCE	367
A.2 NORMAL DISTRIBUTION AND THE CENTRAL LIMIT THEO- REM	370
A.3 CHI-SQUARE DISTRIBUTION, STUDENT'S t -DISTRIBUTION, AND CONFIDENCE INTERVALS	373
A.3.1 Chi-Square Distribution	373
A.3.2 Student's t -Distribution	375
A.3.3 Confidence Intervals	376
A.4 NOTES & SELECTED BIBLIOGRAPHY	377
REFERENCES	379
INDEX	439