

CONTENTS

| | |
|----------------------------|-----|
| Preface | vii |
| Acknowledgements | ix |

CHAPTER I

STATIC MAXIMAL FLOW

| | |
|--|----|
| Introduction | 1 |
| 1. Networks | 2 |
| 2. Flows in networks | 4 |
| 3. Notation | 9 |
| 4. Cuts | 10 |
| 5. Maximal flow | 11 |
| 6. Disconnecting sets and cuts | 14 |
| 7. Multiple sources and sinks | 15 |
| 8. The labeling method for solving maximal flow problems | 17 |
| 9. Lower bounds on arc flows | 22 |
| 10. Flows in undirected and mixed networks | 23 |
| 11. Node capacities and other extensions | 23 |
| 12. Linear programming and duality principles | 26 |
| 13. Maximal flow value as a function of two arc capacities | 30 |
| References | 35 |

CHAPTER II

FEASIBILITY THEOREMS AND COMBINATORIAL APPLICATIONS

| | |
|--|----|
| Introduction | 36 |
| 1. A supply-demand theorem | 36 |
| 2. A symmetric supply-demand theorem | 42 |
| 3. Circulation theorem | 50 |
| 4. The König-Egerváry and Menger graph theorems | 53 |
| 5. Construction of a maximal independent set of admissible cells | 55 |
| 6. A bottleneck assignment problem | 57 |
| 7. Unicursal graphs | 59 |
| 8. Dilworth's chain decomposition theorem for partially ordered sets | 61 |
| 9. Minimal number of individuals to meet a fixed schedule of tasks | 64 |
| 10. Set representatives | 67 |

CONTENTS

| | | |
|-----|--|----|
| 11. | The subgraph problem for directed graphs | 75 |
| 12. | Matrices composed of 0's and 1's | 79 |
| | References | 91 |

CHAPTER III

MINIMAL COST FLOW PROBLEMS

| | |
|--|-----|
| Introduction | 93 |
| 1. The Hitchcock problem | 95 |
| 2. The optimal assignment problem | 111 |
| 3. The general minimal cost flow problem | 113 |
| 4. Equivalence of Hitchcock and minimal cost flow problems | 127 |
| 5. A shortest chain algorithm | 130 |
| 6. The minimal cost supply-demand problem: non-negative directed cycle costs | 134 |
| 7. The warehousing problem | 137 |
| 8. The caterer problem | 140 |
| 9. Maximal dynamic flow | 142 |
| 10. Project cost curves | 151 |
| 11. Constructing minimal cost circulations | 162 |
| References | 169 |

CHAPTER IV

MULTI-TERMINAL MAXIMAL FLOWS

| | |
|--|-----|
| Introduction | 173 |
| 1. Forests, trees, and spanning subtrees | 173 |
| 2. Realization conditions | 176 |
| 3. Equivalent networks | 177 |
| 4. Network synthesis | 187 |
| References | 191 |