OptimizationCity

ENCYCLOPEDIA OF OPERATIONS RESEARCH

Course 1: Modeling Linear Programming

Author: OptimizationCity Group

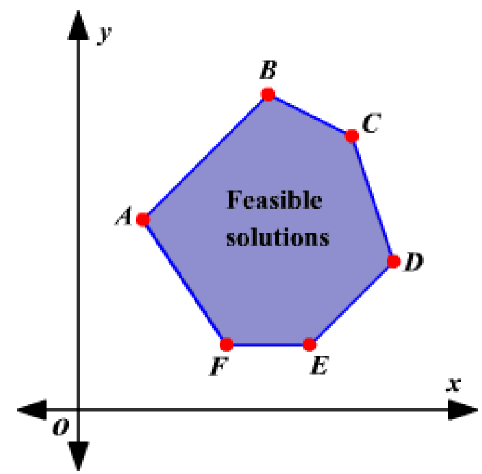
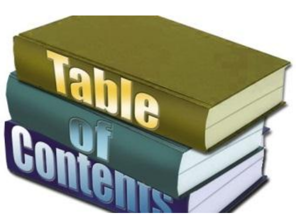


Table of content



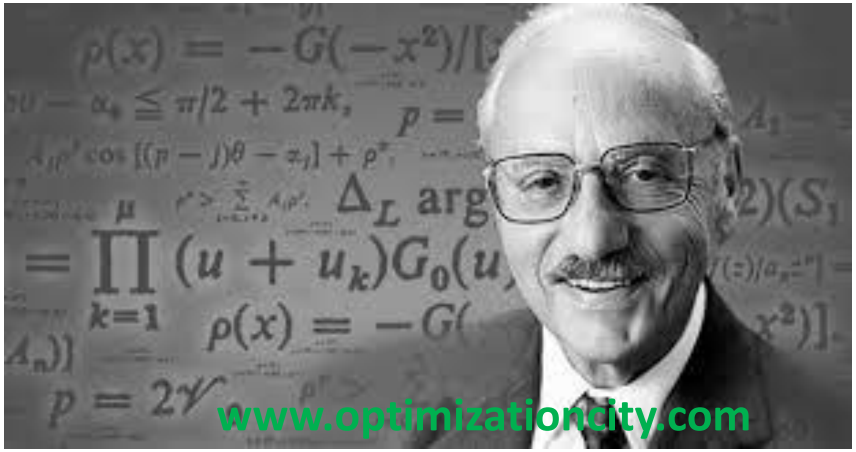
What is Operations Research

What is Operations Research?

Operations research is a scientific approach to the decisions that are made during the operation of organized systems. In other words, operations research is used in issues related to the guidance and coordination of operations and various activities. Operations research is used in a variety of fields such as economics, trade, industry, government, health, and so on. Operations research examines issues with a systematic approach and tries to resolve the conflict of resources between different parts of an organization in such a way that the best result or the optimal answer is created for the whole organization.

What is Operations Research

Operations research includes various sections, of which mathematical linear programming is one of the most important. In linear programming, a mathematical model is used to explain the problem. The word linear means that all mathematical relations of this model must necessarily be linear. The birth of linear programming dates back to 1947, when George Dantzig invented the simplex method for solving general linear programming problems for the optimal programming computation project. The invention was so influential in management science that it led George Dantzig to the Nobel Prize but never won it.



What is Operations Research

Formulating linear programming

Example: A door and window manufacturing company has three plants. plant 1 produces aluminum frames and metal parts. In plant 2, wooden frames are produced; and in plant 3, the glass is cut and mounted on the frames. The manager of these workshops is looking to produce two new products. He/she has asked the operations research center to increase the production of each product according to the capacity of the plants. Product 1 is a door with an aluminum frame and product 2 is glass windows with a wooden frame. Due to the fact that both products require plant 3 for welding, the capacity of this plant will lead the competition between these two products. The table below shows all data you need.

Formulating linear programming

Production time per batch, Hours

product / plant	1	2	Production time available per week, Hours
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch,$	**3**	**5**	

Formulating linear programming

x_1 and x_2 represent the number of products 1 and 2 per hours, respectively, and z represents the profit from sales per hour. In the operations research literature, x_1 and x_2 are called decision variables and z is called an objective function.

$$Max \quad Z = 3x_1 + 5x_2$$

Maximizing profit from sales

$$s.t.$$

$$x_1 \leq 4$$

Production capacity of plant 1

$$2x_2 \leq 12$$

Production capacity of plant 2

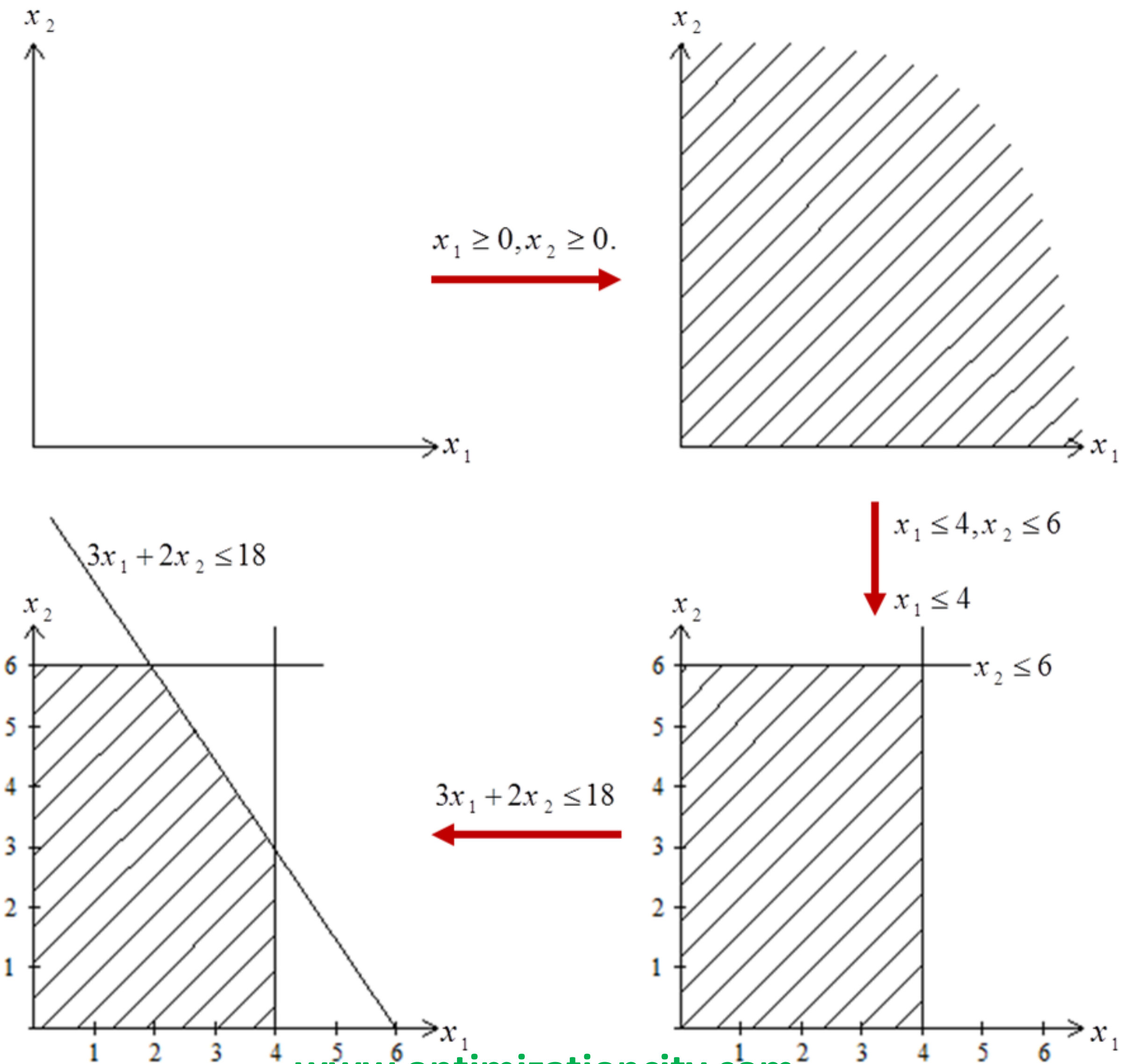
$$3x_1 + 2x_2 \leq 18$$

Production capacity of plant 3

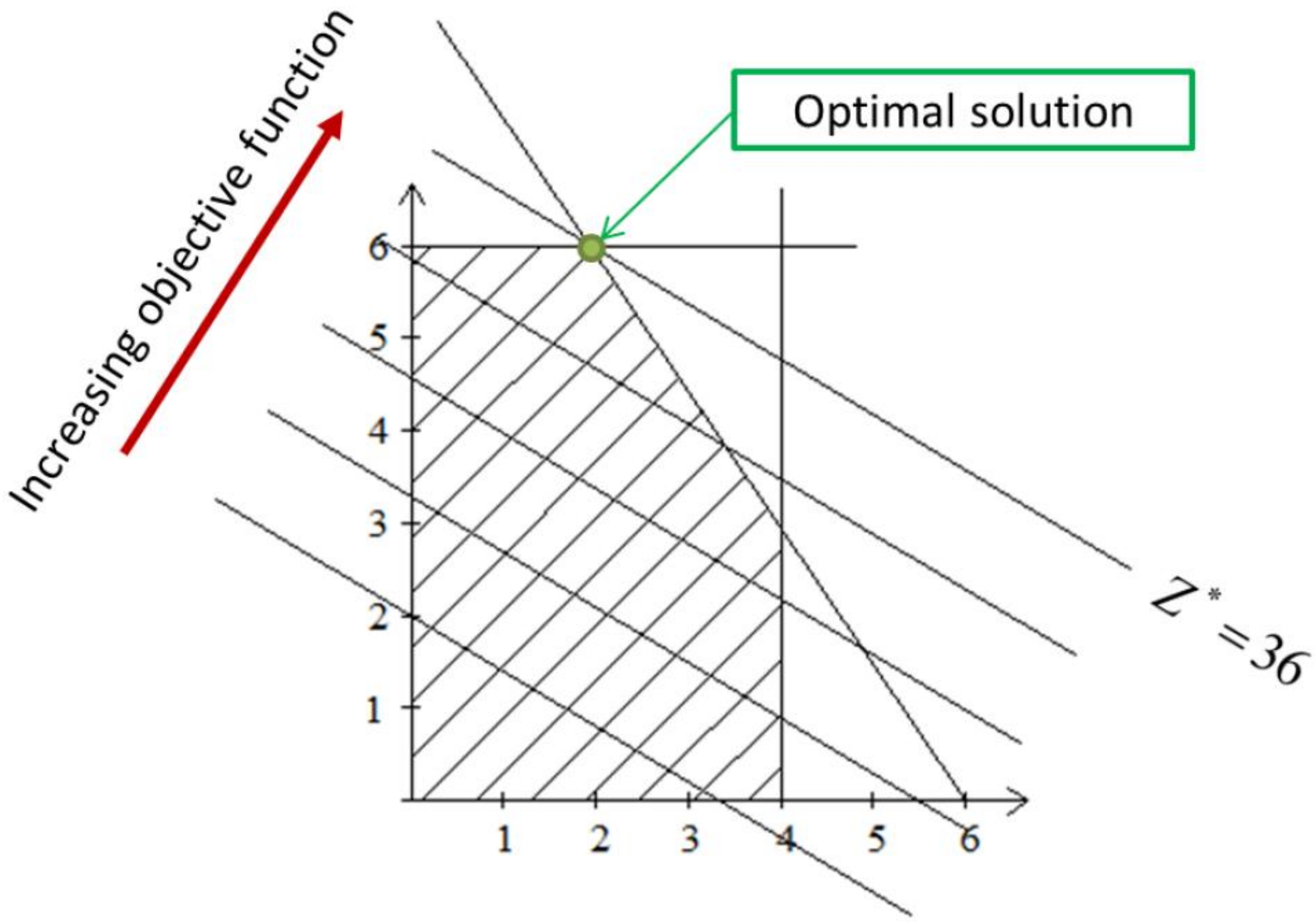
$$x_1 \geq 0, x_2 \geq 0.$$

Non-negativity of decision variables

Formulating linear programming



Formulating linear programming



Formulating linear programming

Product / Source	Consumption of resources per unit of product 1	2	. . .	n	Available value of sources
1	a_{11}	a_{12}	. . .	a_{1n}	b_1
2	a_{21}	a_{22}	. . .	a_{2n}	b_2
⋮	⋮	⋮	⋮	⋮	⋮
m	a_{m1}	a_{m2}	. . .	a_{mn}	b_m
The amount of change in Z per unit increase in product	c_1	c_2	. . .	c_n	
Production amount of the product	x_1	x_2	. . .	x_n	

Mathematical representation of linear model

According to the table above, the general form resource allocation problem can be expressed in mathematical form as follows.

$$
\begin{aligned}
&Max \quad Z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \\
&s.t. \\
&\qquad a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \le b_1 \\
&\qquad a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n \le b_2 \\
&\qquad \qquad \vdots \\
&\qquad a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \le b_m \\
&\qquad x_1 \ge 0, x_2 \ge 0, \cdots, x_n \ge 0.
\end{aligned}
$$

The above standard model is called the ***linear programming problem***.

Terms of linear programming

For simplicity in expressing problems in linear programming, common words are defined as follows.

Objective function: Refers to the function $c_1x_1+c_2x_2+...+c_nx_n$ that is intended to find the maximum.

Functional constraint: refers to a function that indicates the total consumption of each resource for all products.

Non-negativity constraint: Refers to x_j>=0 $x_j \geq 0$.

Constraint: Refers to the set of functional constraints and Non-negativity.

Decision variable: Refers to the variable x_j that indicate the amount of each product.

Parameter: Refers to data a_{ij},b_i,c_j

Terms of linear programming

Note: The standard model may differ from linear programming models in reality. Other forms of linear programming problem that can be converted to standard form are as follows:

- ✓ Instead of maximizing (Max), minimizing (Min) may be considered.
- ✓ Some functional constraints may be greater than equal(≥):

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \geq b_i$$

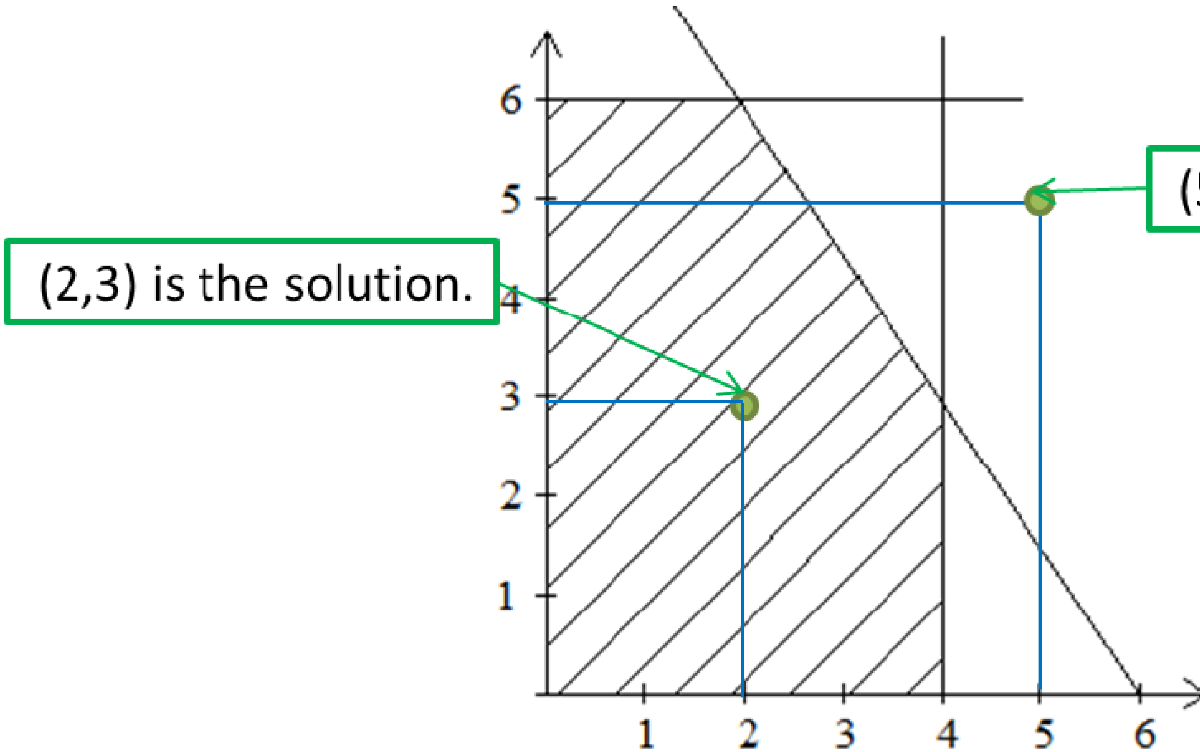
- ✓ Some functional constraints may be equal:

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n = b_i$$

Some variables may be only negative; or either negative or positive (unrestricted in sign).

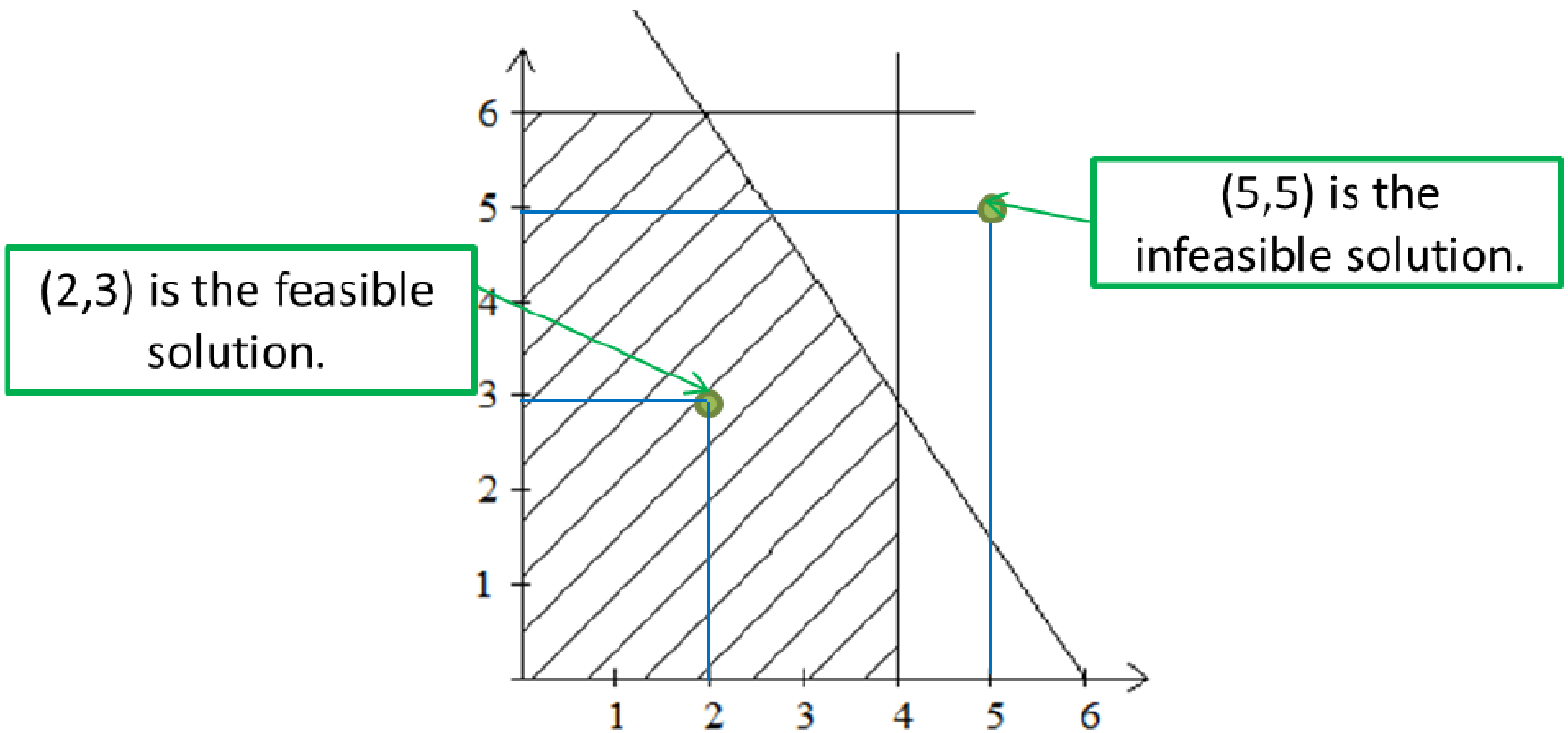
Terms of linear programming

Solution: Refers to any set of values assigned to decision variables ($x_1, x_2, \ldots, x_n$). For example, in the previous example, points (2,3) and (5,5) are the answers of the linear programming model, and it does not matter if they are inside or outside the feasible region.



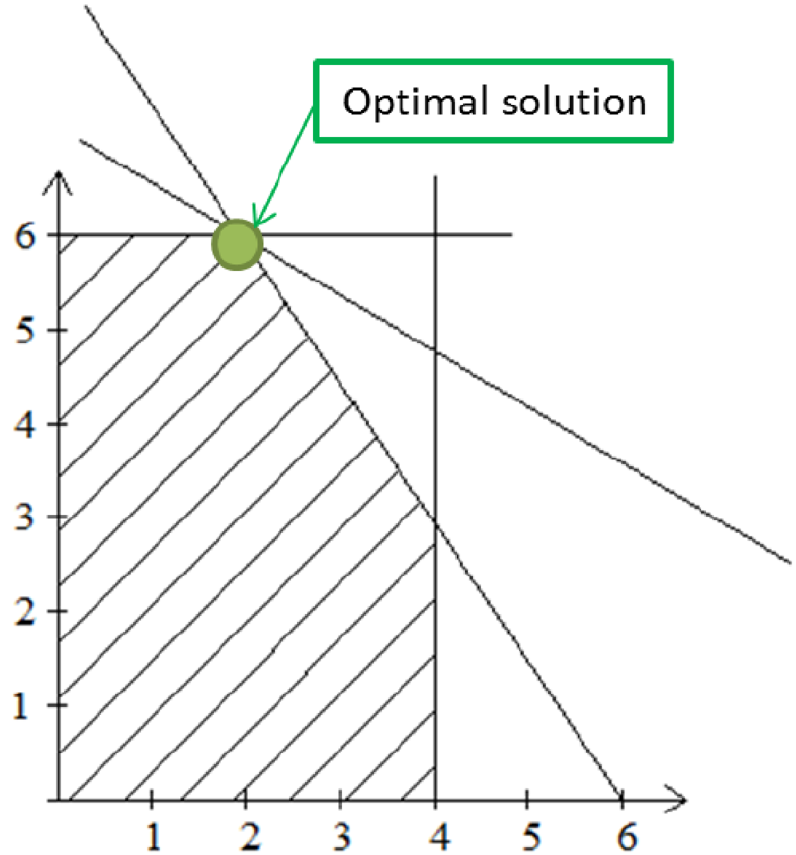
Terms of linear programming

Feasible solution: Refers to the solution that applies to all constraints (functional and non-negative). For example, in the figure above, point (2,3) is a feasible solution and solution (5,5) is an infeasible solution.



Terms of linear programming

Optimal solution: A feasible solution for which the value of the objective function is the most desirable. The following figure shows the optimal solution to the previous example.



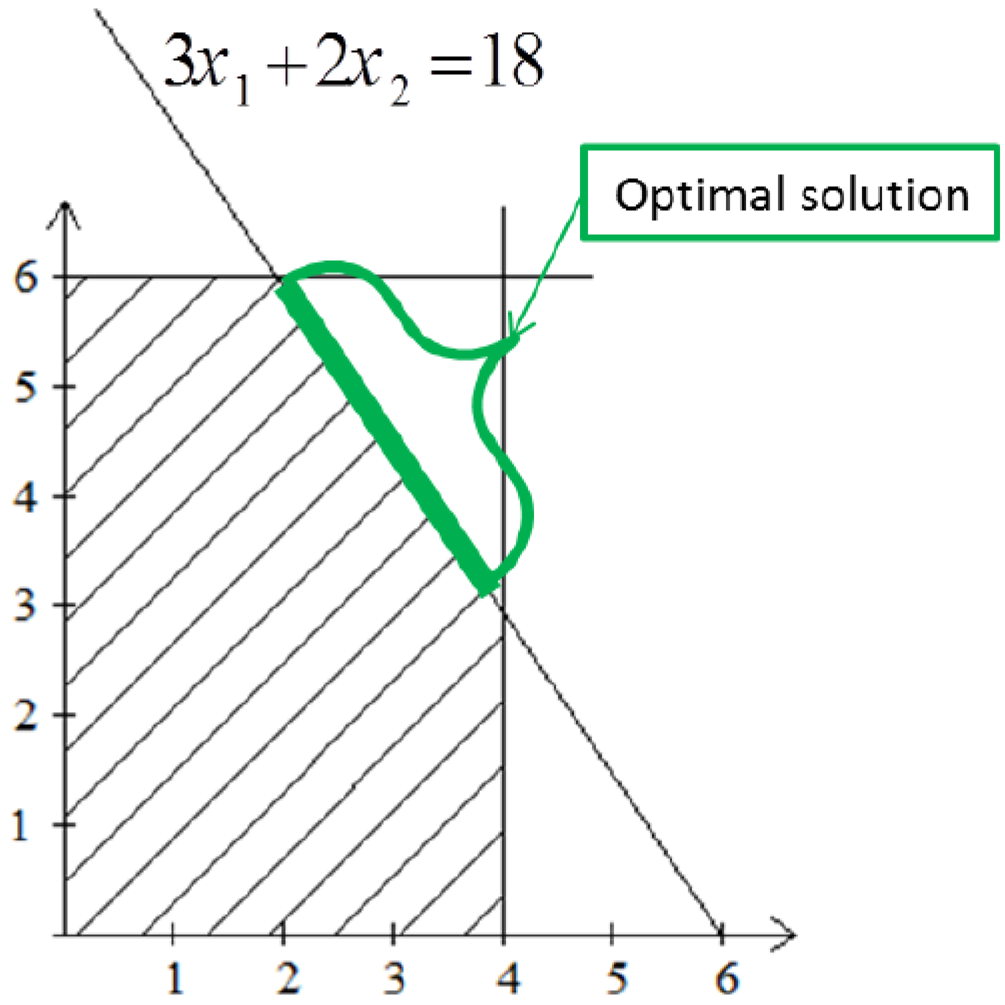
Terms of linear programming

There are four possibilities for the optimal solution:

1- **Unique optimal solution**: In this case, the optimal solution is only one point of feasible region (similar to the figure above).

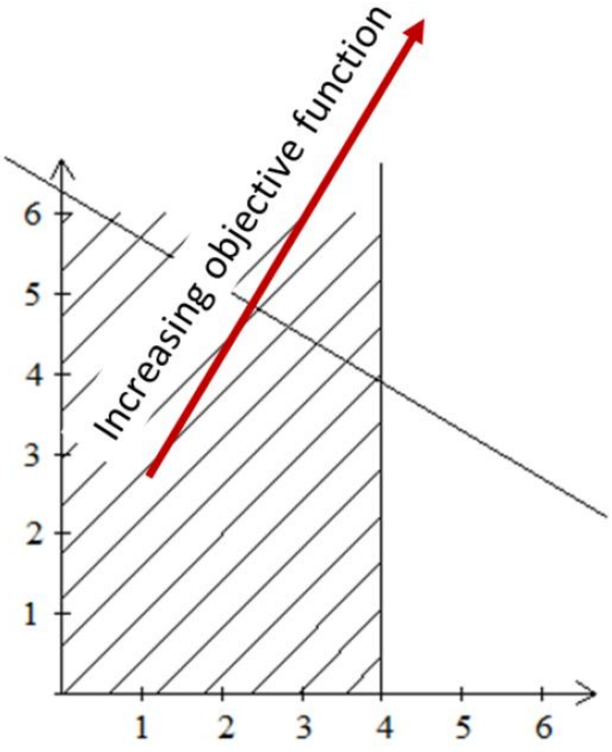
2- **Multiple (alternative) optimal solution**: Often the linear programming problem has a single solution. But this is not necessarily the case. In the previous example, if the profitability of product 2 decreases from 5 to 2 units, then the line of the objective function will be parallel to the line $3x_1 + 2x_2 = 18$, and therefore all points on the line between points (2,6) and (4,3) are the optimal solution.

Terms of linear programming



Terms of linear programming

3-**Unbound solution**: This solution occurs when the constraints can not prevent the infinite increase of the objective function (*Z*) in the desired direction. In the previous example, if both constraints $2x_2 <= 12$ and $3x_1+2x_2 <= 18$ are omitted, the possible space becomes as follows where the value of the objective function can be increased infinitely.



4- **Infeasible solution**: In this case, the problem has no feasible solution or the feasible region is empty. In the previous example, if the constraint $2x_2<=12$ changes to $2x_2>=12$ and the constraint $x_1<=4$ changes to $x_1>=4$, the feasible region is empty and the problem has no solution.

5- **Degeneracy**

In a two-variable problem, if there is a corner point of the intersection of more than two constraints, it is a degeneracy problem (this problem can be used for n-variable problems). To clarify the issue, consider the following example:

Terms of linear programming

Example: Consider this problem:

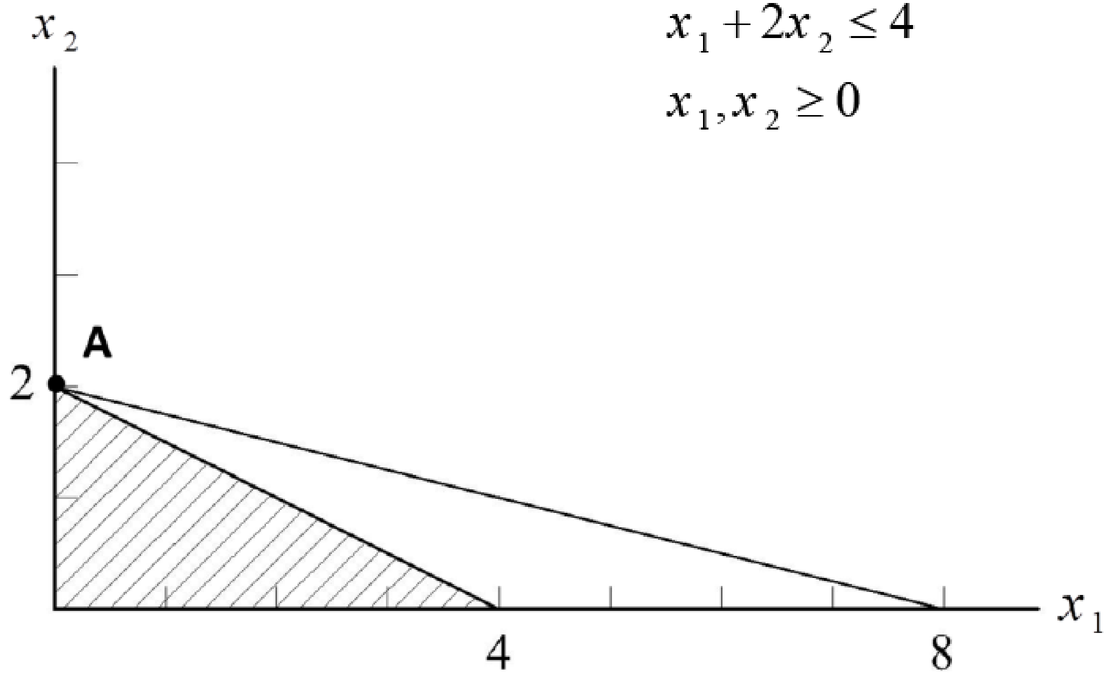
$$Max\ Z = 3x_1 + 9x_2$$

$$s.t.$$

$$x_1 + 4x_2 \leq 8$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$



Terms of linear programming

In this problem, it can be seen that at point A, three corner points have coincided. The points are:

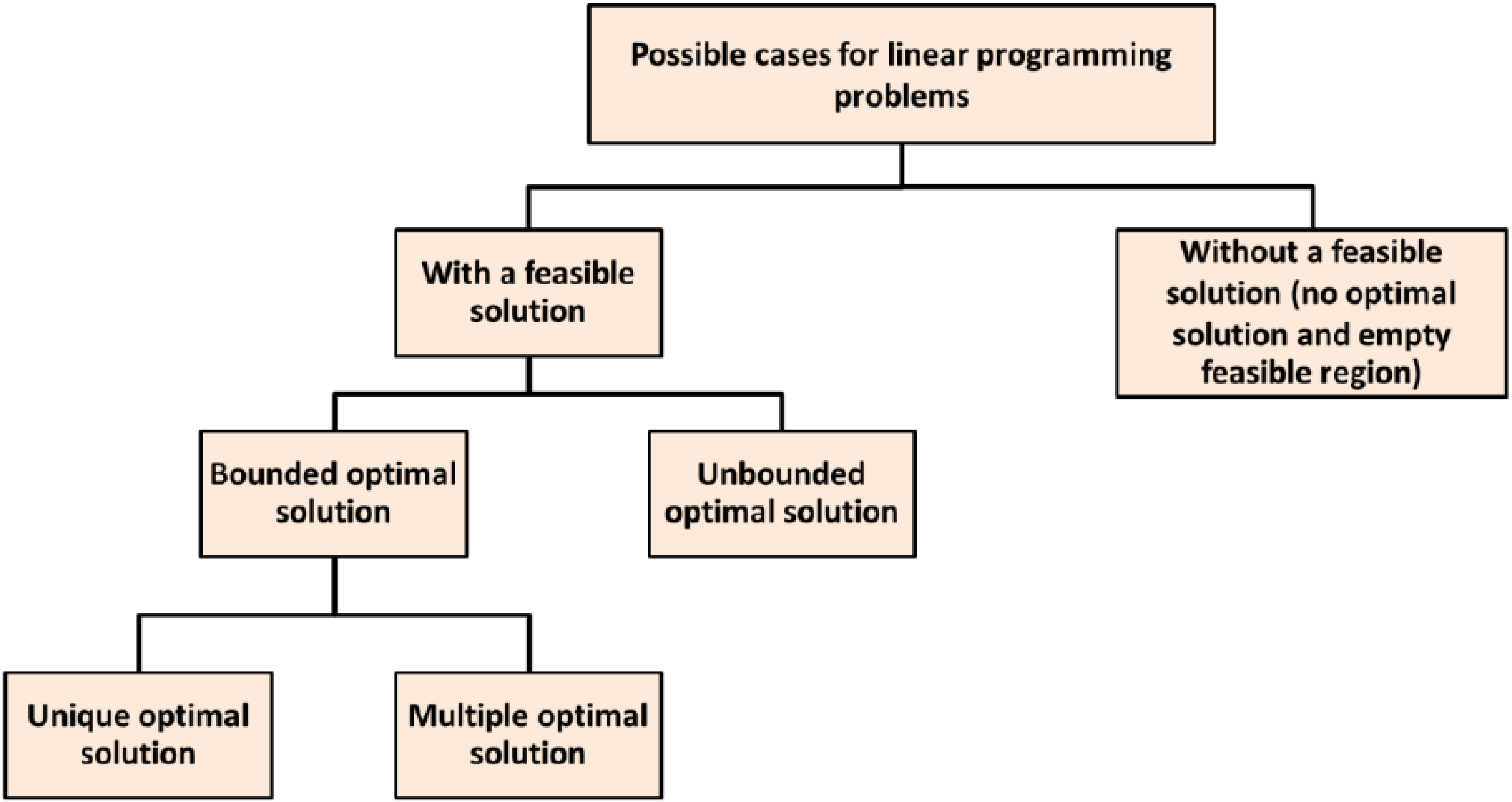
1) The point resulting from the intersection of the constraints corresponding to the first constraint with the x_2 axis

2) The point resulting from the intersection of the constriants corresponding to the second constraint with the x_2 axis

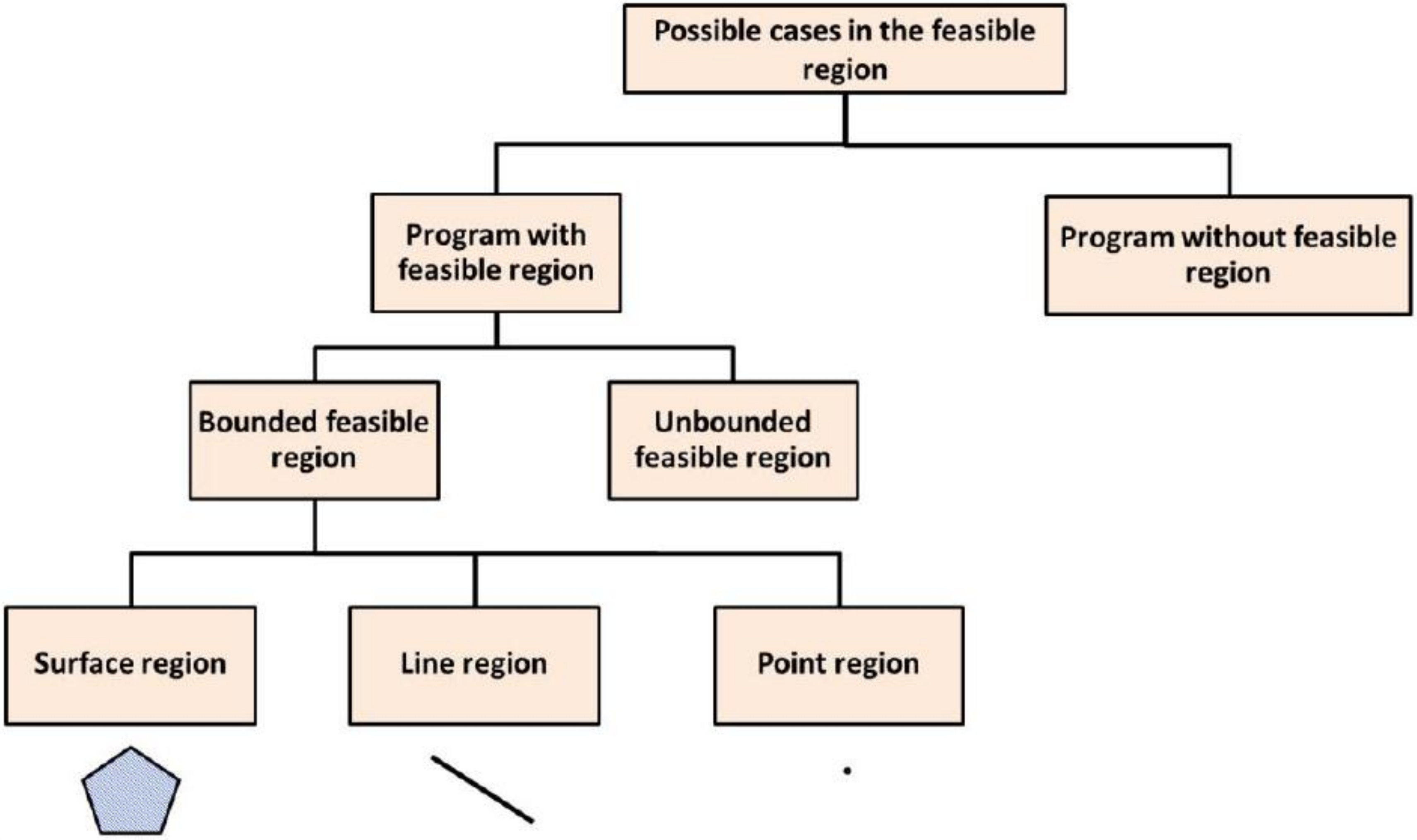
3) The point resulting from the intersection of the constraints corresponding to both constraints

Later, it is said that in the simplex method, each iteration actually represents the characteristics of a corner point, and since several corner points coincide in degeneracy, we may encounter a situation where in successive iterations on these points.

Classification of linear programming



Classification of linear programming



Classification of constraints in linear programming

According to the effect of the constraint on the formation of the feasible region and how the optimal solution is found on them, the constraints can be classified into two groups.

Effective constraints

There are constraints that are effective in forming the feasbile area, and adding any new effective limitation to the model causes a decrease in the justified area, and removing the effective restriction causes an increase in the justified area.

Redundant constraints

There are restrictions that do not affect the creation of the feasbile area, and their presence or absence does not change the feasbile area.

Exercise

Example: Consider this problem.

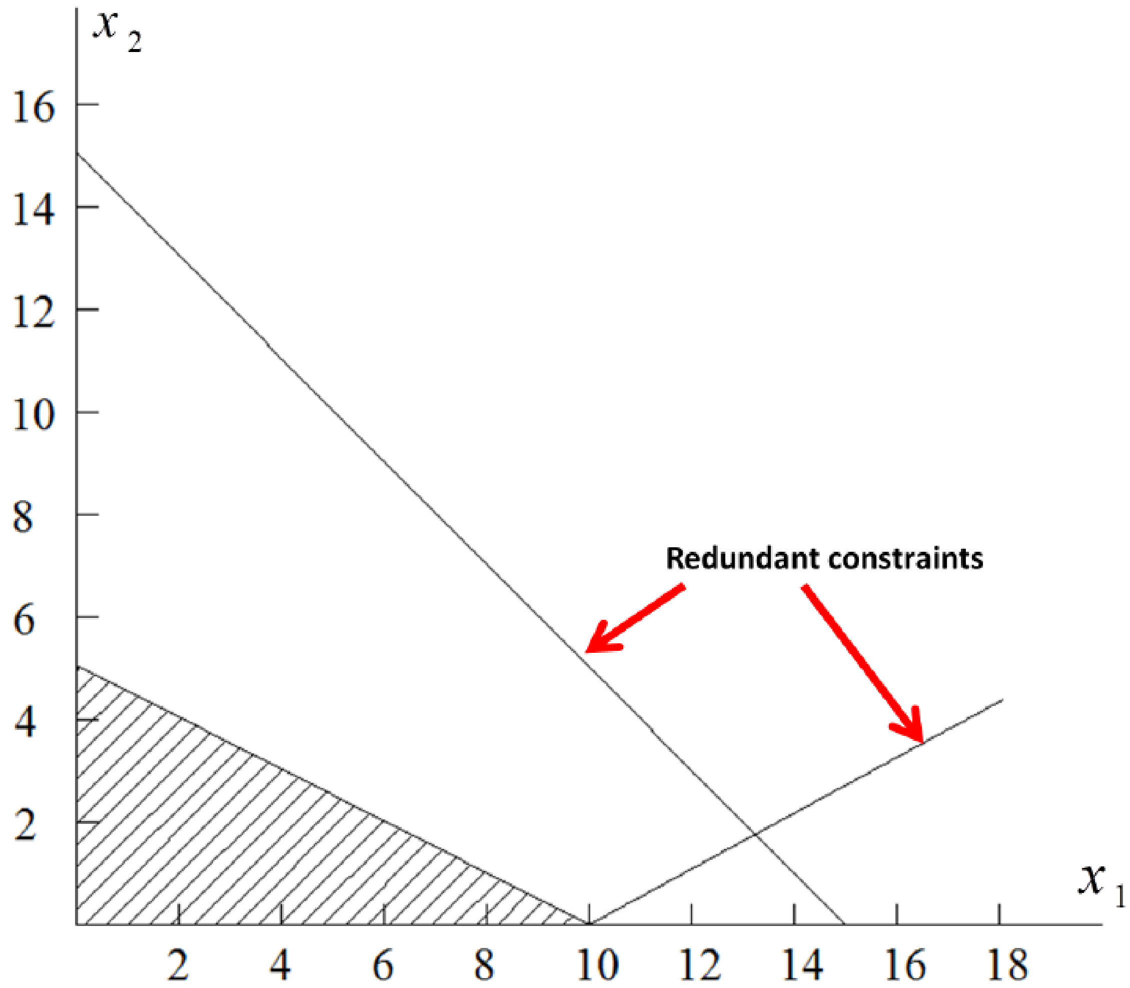
$$Max\ Z = 6x_1 + 12x_2$$

$$s.t. \quad x_1 + 2x_2 \le 10$$

$$2x_1 - 5x_2 \le 20$$

$$x_1 + x_2 \le 15$$

$$x_1, x_2 \ge 0$$



Exercise

Example: An area consists of three farms with two limitations, land and water, that limit the possibilities of planting these farms. Information on the water alocation and usable land of the three farms is given in the table below.

Farm	Usable land (Acres)	Water allocation (Acres Feet)
1	400	600
2	600	800
3	300	375

Exercise

The crops suited for this region include sugar beets, cotton, and sorghum. These crops differ primarily in their expected net return (profit) per acre and their consumption of water. In addition, the maximum acreage that can be devoted to each of these crops is set.

Crop	Net return (Dollar/Acre)	Water consumption (Acre Feet/Acre)	Maximum land for farming (Acres)
Sugar beets	400	3	600
Cotton	300	2	500
Sorghum	100	1	325

Exercise

Exercise

To ensure equity between the three farms, it has been agreed that every famer will plant the same proportion of its available land. For example, if farm 1 plants 200 of its available 400 acres, then farm 2 must plant 300 of its 600 acres, while farm 3 plants 150 acres of its 300 acres. However, any combination of the crops may be grown at any of the farm.

The job is to plan how many acres to devote to each crop at the farms while satisfying the given restrictions. The objective is to maximize the total net return as a whole.

Exercise

Solution: We are going to formalate this problem as a linear programming model. The quantities to be decided are the number of acres to devote to each of the three crops at each of the three farms, which are the decision variables as follows.

Crop \ Farm	1	2	3
Sugar beets	x_1	x_2	x_3
Cotton	x_4	x_5	x_6
Sorghum	x_7	x_8	x_9

Exercise

$\text{Max } Z=400(x_1+x_2+x_3)+300(x_4+x_5+x_6)+100(x_7+x_8+x_9)$

Subject to the following constraints:

1- Usable land for each farm(acre):

Farm 1: $x_1+x_4+x_7<=400$

Farm 2: $x_2+x_5+x_8<=600$

Farm 3: $x_3+x_6+x_9<=300$

2- Water allocation for each farm:

Farm 1: $3x_1+2x_4+x_7<=600$

Farm 2: $3x_2+2x_5+x_8<=800$

Farm 3: $3x_3+2x_6+x_9<=375$

3- Total acreage for each crop:

Crop 1: $x_1+x_2+x_3<=600$

Crop 2: $x_4+x_5+x_6<=500$

Crop 3: $x_7+x_8+x_9<=325$

Exercise

4- Equal proportion of land planted:

Equality of cultivated land to usable land for farms 1 and 2:

$(x_1+x_4+x_7)\div400=(x_2+x_5+x_8)\div600$

Equality of cultivated land to usable land for farms 1 and 3:

$(x_1+x_4+x_7)\div400=(x_3+x_6+x_9)\div300$

Equality of cultivated land to usable land for farms 2 and 3:

$(x_2+x_5+x_8)\div600=(x_3+x_6+x_9)\div300$

The above equations are not yet in an appropriate form for a linear programming model because some of the variables are on the right-hand side. Hence, their final form is:

$3(x_1+x_4+x_7)-2(x_2+x_5+x_8)=0$

$4(x_3+x_6+x_9)-3(x_1+x_4+x_7)=0$

$(x_2+x_5+x_8)-2(x_3+x_6+x_9)=0$

Exercise

5- Non-negativity of decision variables is as follows.

$x_j >= 0 \quad \forall j = 1, \ldots, 9$

The optimal solution to the above linear programming model is as follows:

Crop \ Farm	1	2	3
Sugar beets	25	100	133.3
Cotton	150	250	100
Sorghum	0	0	0

Exercise

Example: A Company has discontinued the production of a certain unprofitable product line, which creating the excess capacity on machines. The management is considering devoting this excess capacity to products; products 1, 2, and 3. The available capacity on the machines is summarized in the following table:

Machine Type	Available time (Machine hours per week)
Milling machine	500
Lather machine	350
Grinder machine	150

Exercise

Exercise

The number of machine-hour required for each unit of the products is:

Machine type	Product 1	Product 2	Product 3
Milling machine	9	3	5
Lather machine	5	4	0
Grinder machine	3	0	2

Exercise

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and the sales potential for product 3 is 20 units per week. The unit profit would be \$50, \$20, and \$25, respectively, on products 1, 2, and 3. The objective is to determine how much of each product should produce to maximize profit. Formulate a linear programming model for this problem.

Exercise

Solution:

Let x_i be the number of units of product i produced for $i = 1, 2, 3$. The linear programming model of this problem is as follows.

$$Max\ Z = 50x_1 + 20x_2 + 25x_3$$

$s.t.$

$$\begin{aligned}
&(1) \quad 9x_1 + 3x_2 + 5x_3 \le 500 \\
&(2) \quad 5x_1 + 4x_2 \le 350 \\
&(3) \quad 3x_1 + 2x_3 \le 150 \\
&(4) \quad x_3 \le 20 \\
&\quad x_i \ge 0 \quad i = 1, \ldots, 3.
\end{aligned}$$

Exercise

Constraint 1 is the same as milling machine capacity, constraint 2 is the same as lather machine capacity, constraint 3 is the same as grinder machine capacity, and constraint 4 is demand constriant.

The optimal solution to the above linear programming model is as follows:

Decision variable	X_1	X_2	X_3	Z
Optimal value	26.19	54.76	20.0	2904.7

Exercise

Example: Consider the following problem, where the value of c_1 has not yet been certain.

$$Max\ Z = c_1 x_1 + x_2$$

$$s.t.$$

$$(1) \quad x_1 + x_2 \le 6$$

$$(2) \quad x_1 + 2x_2 \le 10$$

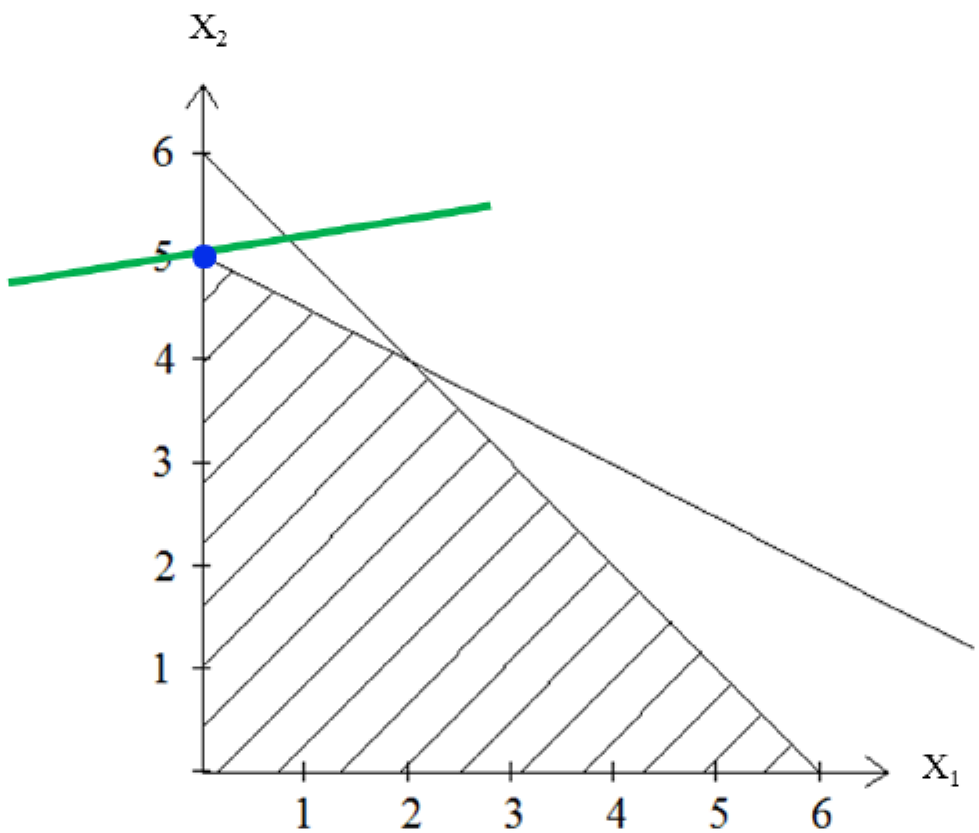
$$x_i \ge 0 \quad i = 1, 2.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1 $-\infty < c_1 < +\infty$.

Exercise

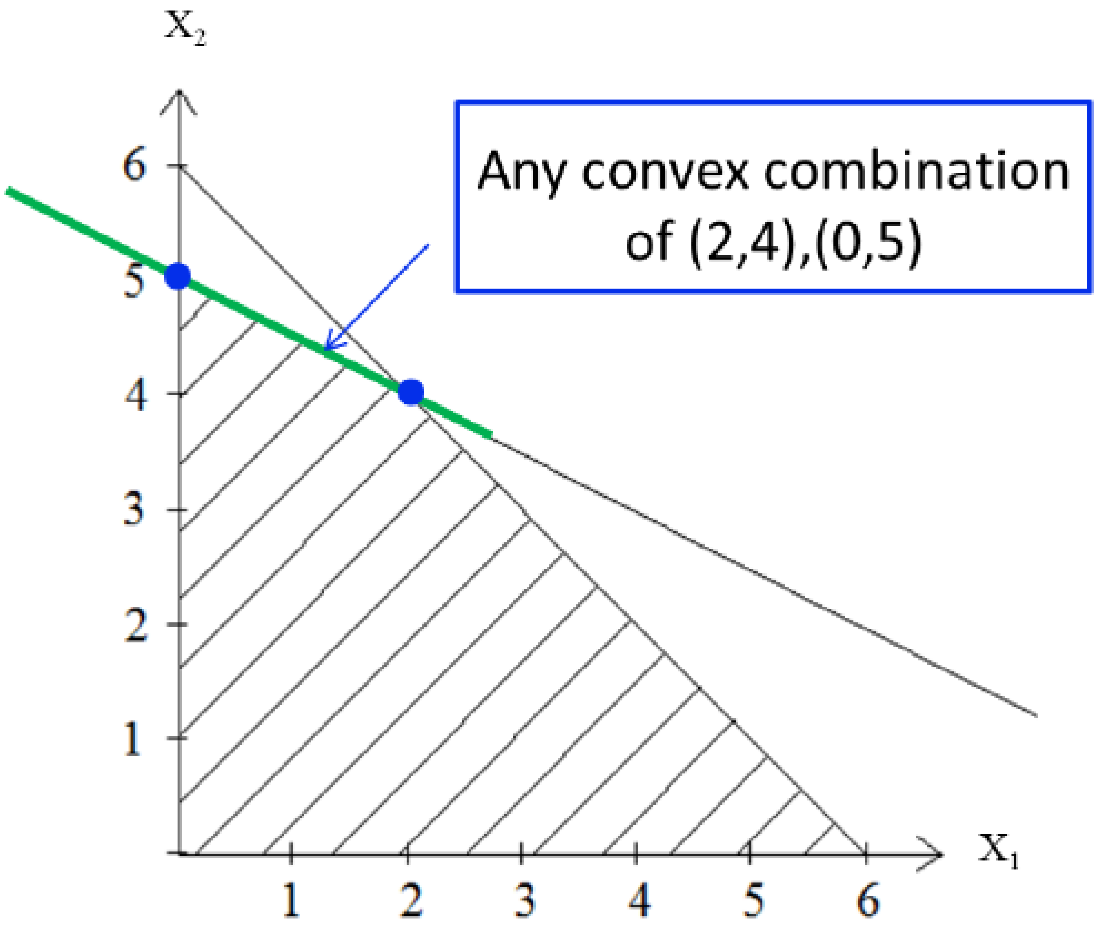
Solution:

$$c < 0.5, x^* = (0, 5), Z^* = 5$$



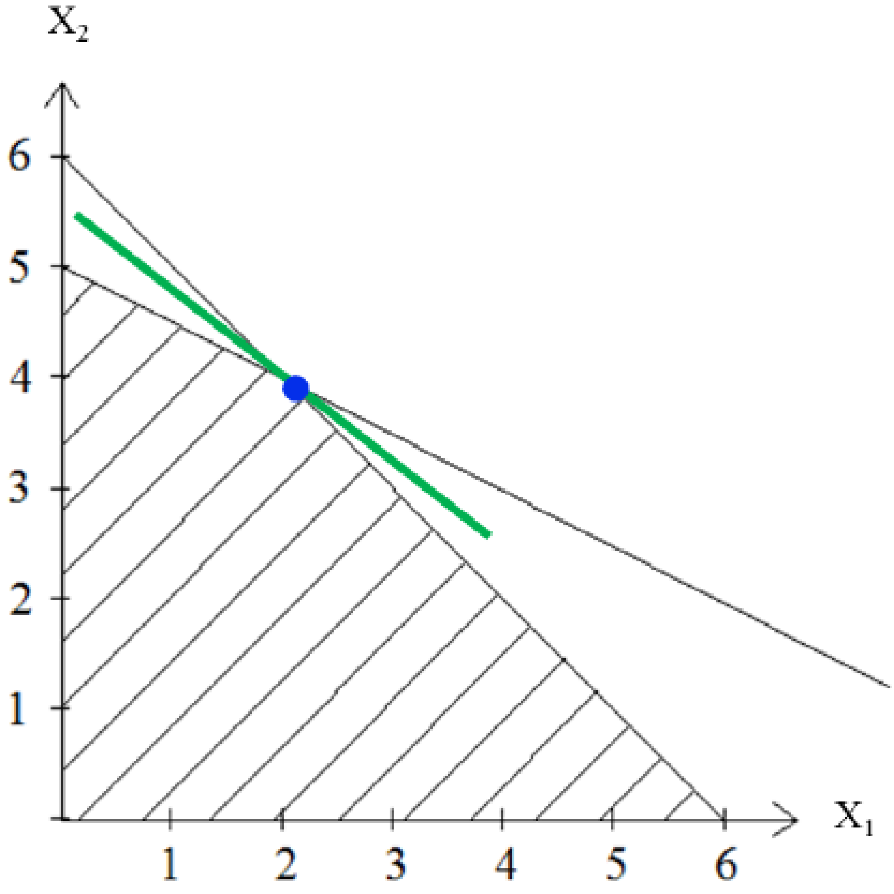
Exercise

$$c = 0.5, x^* = (2,4) \ \& \ (0,5), Z^* = 5$$



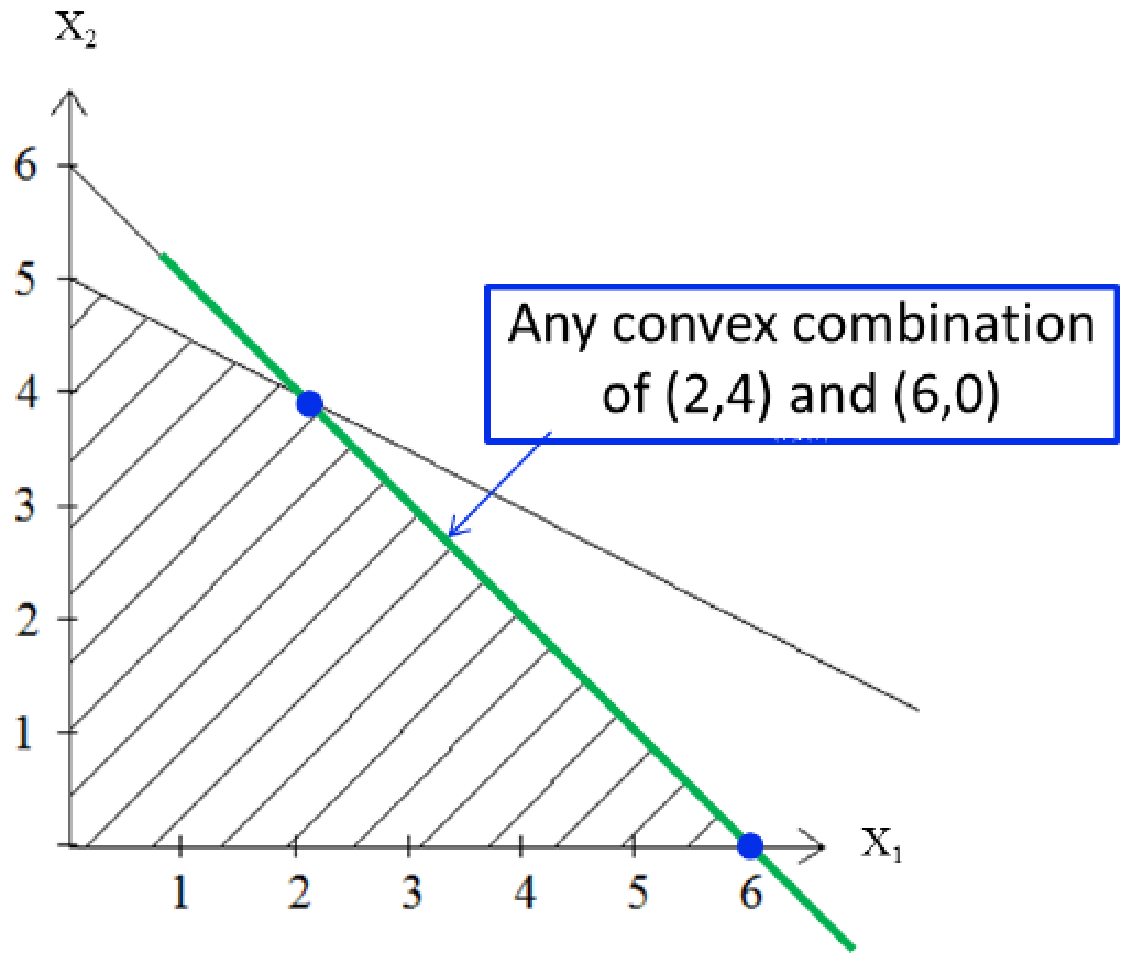
Exercise

$0.5 < c < 1, x^* = (2,4), Z^* = c + 4$



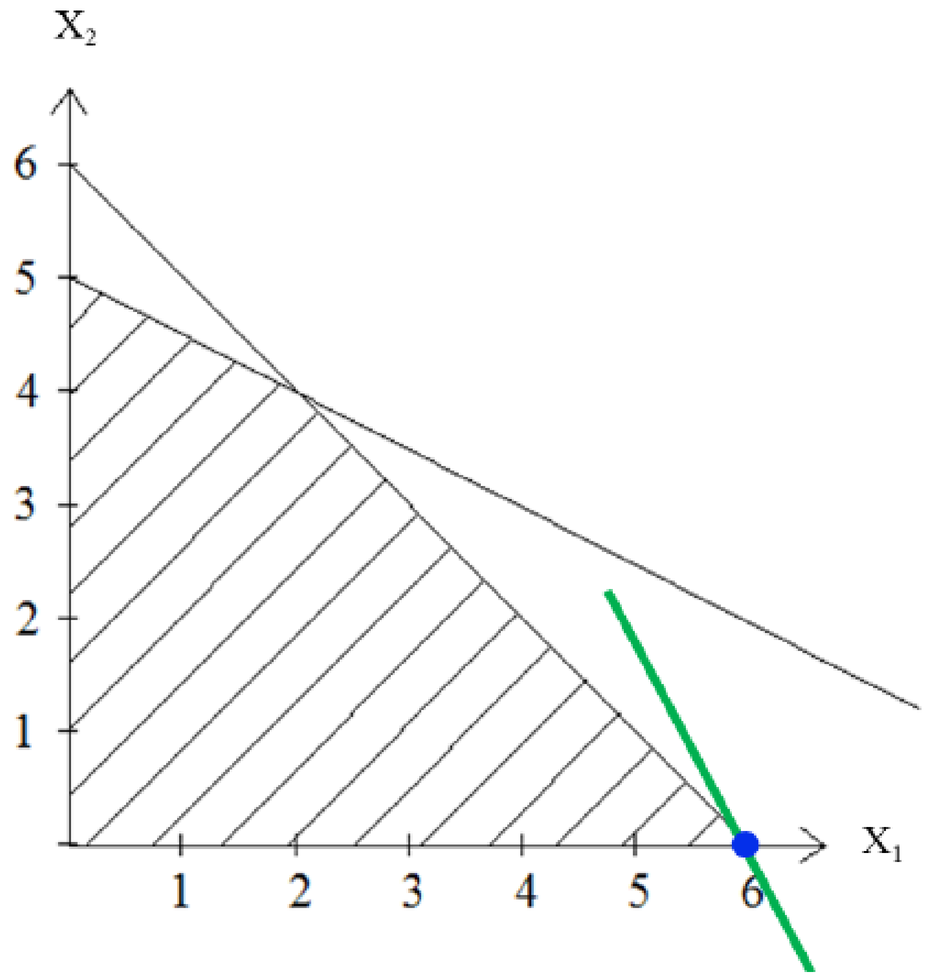
Exercise

$c = 1, x^* = (2,4)\ \&\ (6,0), Z^* = 6$



Exercise

$$c > 1, x^* = (6, 0), Z^* = 6c$$



Exercise

Example: The Company produces precision medical equipment at two factories. Three medical customers have placed orders for products. The table below shows what the cost would be for shipping each unit from each factory to each of these customers. Also shown are the number of units that will be produced at each factory and the number of units ordered by each customer.

To / From	Unit shipping cost			Output of each factory
	Customer 1	Customer 2	Customer 3	
Factory 1	$600	$800	$700	400
Factory 2	$400	$900	$600	500
Order size per month	300	200	400	

A decision now needs to be made about the shipping plan for how many units to ship from each factory to each customer.

Exercise

Solution:

Let X_{ij} be the number of units shipped from factory i (1 and 2) to customer j (1,2 ,and 3).

Objective function: minimizing the shipping cost:

Min $Z=600x_{11}+800x_{12}+700x_{13}+400x_{21}+900x_{22}+600x_{23}$

Output of factory 1:

$x_{11}+x_{12}+x_{13}=400$

Output of factory 2:

$x_{21}+x_{22}+x_{23}=500$

Order of customer 1:

$x_{11}+x_{21}=300$

Order of customer 2:

$x_{12}+x_{22}=200$

Order of customer 2:

$x_{13}+x_{23}=400$

As a result, the model as follow:

Exercise

$$Min\ Z = 600x_{11} + 800x_{12} + 700x_{13} + 400x_{21} + 900x_{22} + 600x_{23}$$

$s.t.$

$$(1) \quad x_{11} + x_{12} + x_{13} = 400$$

$$(2) \quad x_{21} + x_{22} + x_{23} = 500$$

$$(3) \quad x_{11} + x_{21} = 300$$

$$(4) \quad x_{12} + x_{22} = 200$$

$$(5) \quad x_{13} + x_{23} = 400$$

$$x_{ij} \geq 0 \quad i = 1, 2,\ j = 1, 2, 3.$$

The above model has the following optimal solution:

$(x_{11},x_{12},x_{13},x_{21},x_{22},x_{23})$=(0,200,200,300,0,200) and Z*=540000

Exercise

Example: The company desires to blend a new alloy of 40 percent *tin*, 35 percent *zinc*, and 25 percent *lead* from several available alloys having the following properties:

Property	Alloy				
	1	2	3	4	5
Percent of tin	60	25	45	20	50
Percent of zinc	10	15	45	50	40
Percent of lead	30	60	10	30	10
Production cost ($/lb)	77	70	88	84	94

The objective is to determine the proportions of these alloys that should be blended to produce the new alloy at a minimum cost. Formulate the problem as the linear program.

Exercise

Solution: Let x_i be the amount of Alloy i used for 1,…,5.

$$Min\ Z = 77x_1 + 70x_2 + 88x_3 + 84x_4 + 94x_5$$

$$s.t.$$

$$\begin{aligned}
&(1) \quad 60x_1 + 25x_2 + 45x_3 + 20x_4 + 50x_5 = 40\\
&(2) \quad 10x_1 + 15x_2 + 45x_3 + 50x_4 + 40x_5 = 25\\
&(3) \quad 30x_1 + 60x_2 + 10x_3 + 30x_4 + 10x_5 = 35\\
&(4) \quad x_1 + x_2 + x_3 + x_4 + x_5 = 1\\
&\qquad x_i \geq 0 \quad i = 1,\ldots,5.
\end{aligned}$$

The optimal solution of the above problem is as follow:

$(x_1,x_2,x_3,x_4,x_5)=(0.0435,0.2826,0.6739,0,0)$ and $Z^*=\$84.43$

Exercise

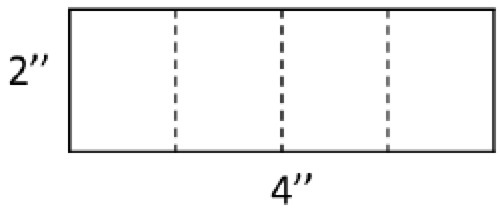
Example: A logging company should prepare orders with the following dimensions and deliver them to the applicants.

Order number	Size of order	The dimensions of the ordered wood
1	1300	$1'' \times 2'' \times 11'$
2	1000	$1'' \times 4'' \times 11'$
3	700	$2'' \times 2'' \times 11'$

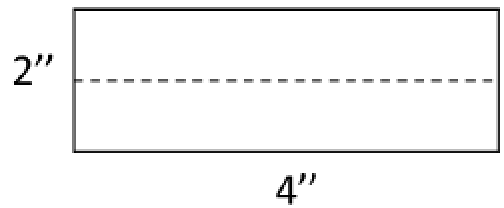
These orders must be made from standard boards with dimensions of 2" x 4" x 11' (2" means 2 inches and 11' means 11 feet). The standard cut to prepare the orders is possible in the following five ways:

Exercise

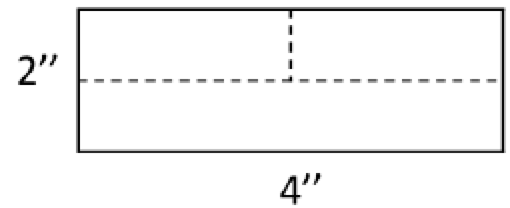
The first way of cutting x_1



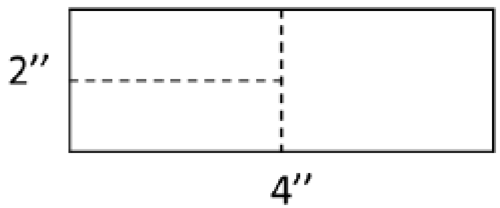
The second way of cutting x_2



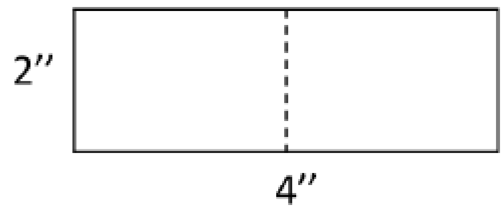
The third way of cutting x_3



The forth way of cutting x_4



The fifth way of cutting x_5



This company intends to use the minimum standard board necessary to prepare orders.

Exercise

Solution:

If x_1, x_2, x_3, x_4, and x_5 are the number of standard boards cut according to one of the five possible ways, the objective function will be as follows:

$$Min\ C = x_1 + x_2 + x_3 + x_4 + x_5$$

Considering that in the cutting of the first type, four boards with the dimensions of the order 1 result, and also in the cutting of the third type and the fourth type, two boards with the dimensions of the order 1 respectively, as a result, the number of order 1 is as follows:

$4x_1+2x_3+2x_4>=1300$

Exercise

Similarly, the constraints related to the other two types of orders are as follows:

Order 2:$2x_2+x_3>=1000$ $2x_2+x_3 \geq 1000$

Order 3: $x_4+2x_5>=750$ $x_4+2x_5 \geq 700$

By adding the condition of variables being non-negative, the model of the above problem can be summarized as follows:

$$Min\ C = x_1 + x_2 + x_3 + x_4 + x_5$$

$$s.t.$$

$$4x_1 + 2x_3 + 2x_4 \geq 1300$$

$$2x_2 + x_3 \geq 1000$$

$$x_4 + 2x_5 \geq 700$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

The optimal solution of the above model is equal to:

$x_1^* = 325, x_2^* = 500, x_3^* = x_4^* = 0, x_5^* = 350, C^* = 1175$ m

Exercise

Example: A manufacturing company has a commitment to deliver 800 refrigerators this year and another 1200 next year to a chain of stores. The production capacity of the company is 1800 refrigerators and the production cost of each unit is estimated at 1000 $ for the current year and 1500 $ for the next year. In addition, every unit produced this year that is not delivered and remains in the warehouse will be charged 200 $.

It is assumed that the manufacturing company does not have any inventory of refrigerators at the beginning of the contract and wants to have no inventory at the end of the second year. This company requests a production plan that minimizes the total cost of fulfilling its contracts.

Exercise

Solution:

We define the decision variables as follows.

t_1 = production quantity for the current year

t_2 = quantity produced for next year

Q_0 = inventory at the beginning of the current year

Q_1 = Balance at the end of the current year

Q_2 = Balance at the end of next year

Because $Q_0=0$ and $Q_2=0$, the objective function becomes as follows:

$$\text{Min } C=1000t_1+200Q_1+1500t_2$$

Exercise

In forming the constraints of the problem, it should be noted that the total inventory at the beginning of the current year plus the production in this year minus the remaining inventory at the end of the year must be equal to the commitment made this year (800), that is, we have:

$$Q_0+t_1-Q_1=800$$

Also, for next year we should have:

$$Q_1+t_2-Q_2=1200$$

Considering that $Q_0=Q_2=0$, the problem model is as follows:

Exercise

$$Min\ C = 1000t_1 + 200Q_1 + 1500t_2$$

$$t_1 - Q_1 = 800$$

$$Q_1 + t_2 = 1200$$

$$t_1 \le 1800$$

$$t_2 \le 1800$$

$$t_1, Q_1, t_2 \ge 0$$

The optimal solution of the above model is equal to

$$t_1^* = 1800, t_2^* = 200, Q_1^* = 1000, C^* = 2,300,000$$

Exercise

Example: Consider the following problem:

$$
\begin{aligned}
Max \quad & Z = 3x_1 + 6x_2 \\
s.t. \quad & x_1 + x_2 \geq 1 \\
& 2x_1 + x_2 \leq 8 \\
& x_1 \leq 3 \\
& x_2 \leq 6 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

a) Solve the above problem.

b) Convert the objective function to Min and solve it again.

c) Change the objective function to Max $Z=x_1$ and solve it again.

d) Change the objective function to Max $Z=-x_1+2x_2$ and solve it again.

e) Change the objective function to Min $Z=x_1-2x_2$ and solve it again.

f) Change the second constraint to equal and solve it again.

g) Change the second constraint to be greater or equal and solve it again.

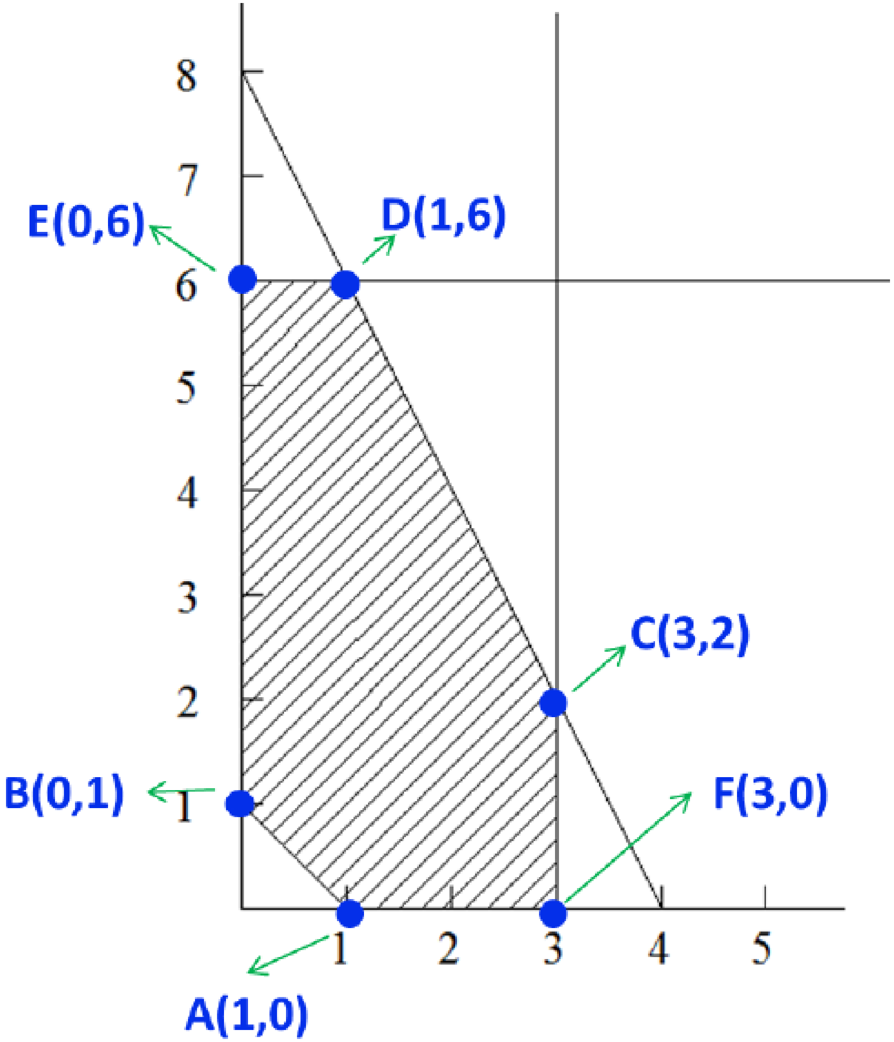
Exercise

Solution:

a)

Considering that, in linear programming problems, the optimal solution will be possible in at least one of the corners of the space, so it is enough to list all corner solutions and the corner solution that has the highest value of the objective function (here the model is maximum) to be selected as the optimal solution. In the figure and table below, these points are specified.

Exercise



Z	(x_1,x_2)	Feasible corner solution
3	(1,0)	A
6	(0,1)	B
21	(3,2)	C
42	(1,6)	D*
36	(0,6)	E
9	(3,0)	F
D* is the optimal solution. $x_1^* = 1, x_2^* = 6, Z^* = 39$		

Exercise

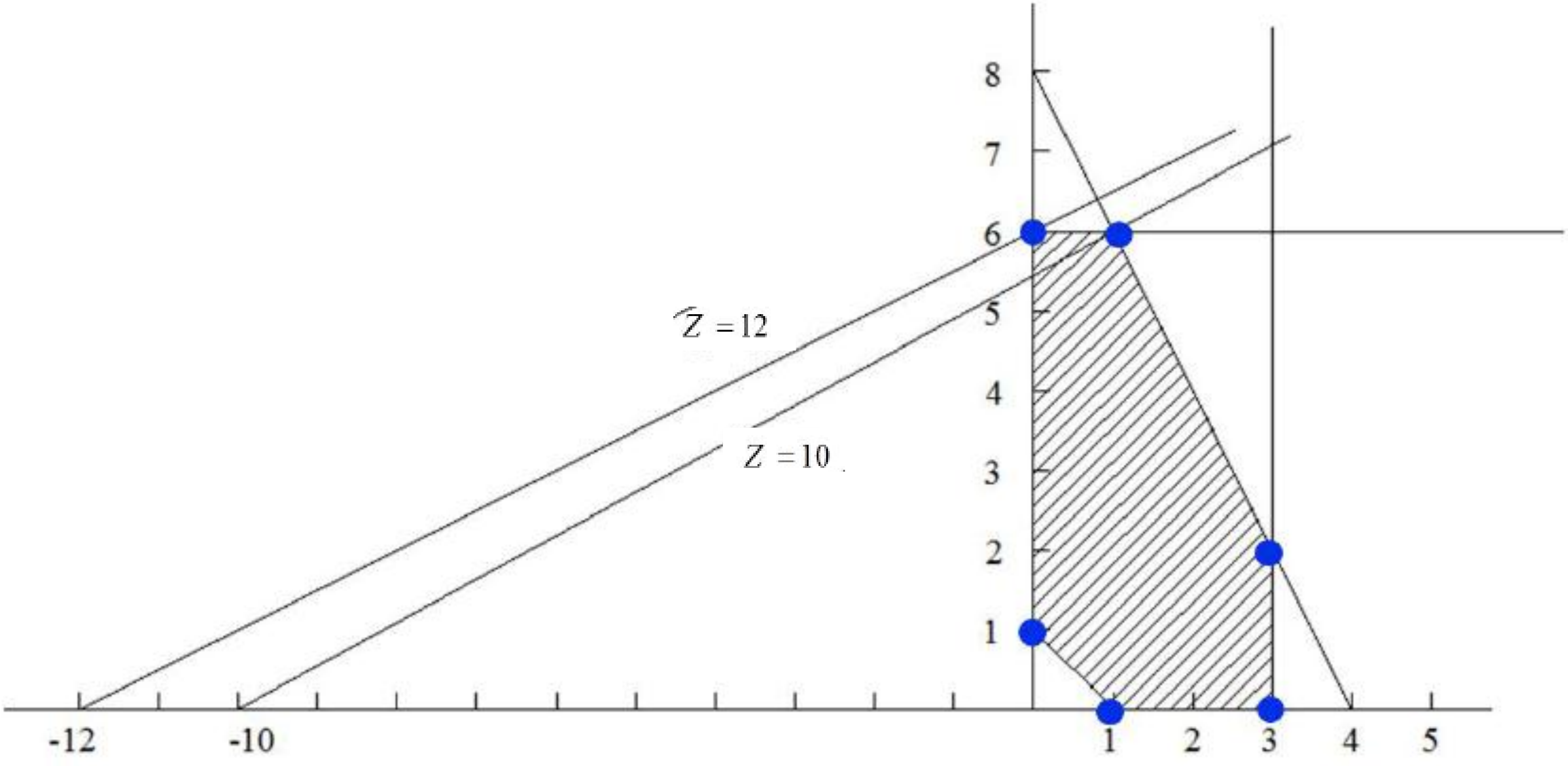
b) According to the figure and feasible point of part (a), point A is optimal.

$$x_1^* = 1, x_2^* = 0, Z^* = 3$$

c) By changing the objective function to Max Z=x_1, the new objective function will be parallel to constraint x_1>=3 and the problem will have a special case of optimal solutions. The corner points C*(3,2) and F*(3,0) are optimal and have value Z*=3.

d) By changing the objective function to Max Z=-x_1+2x_2 , the optimal solution will be E*(0,6) and Z*=12 .

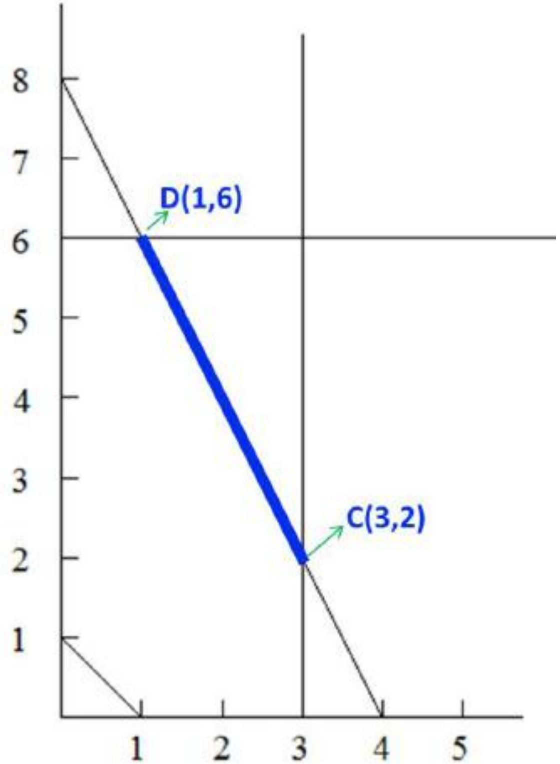
Exercise



Exercise

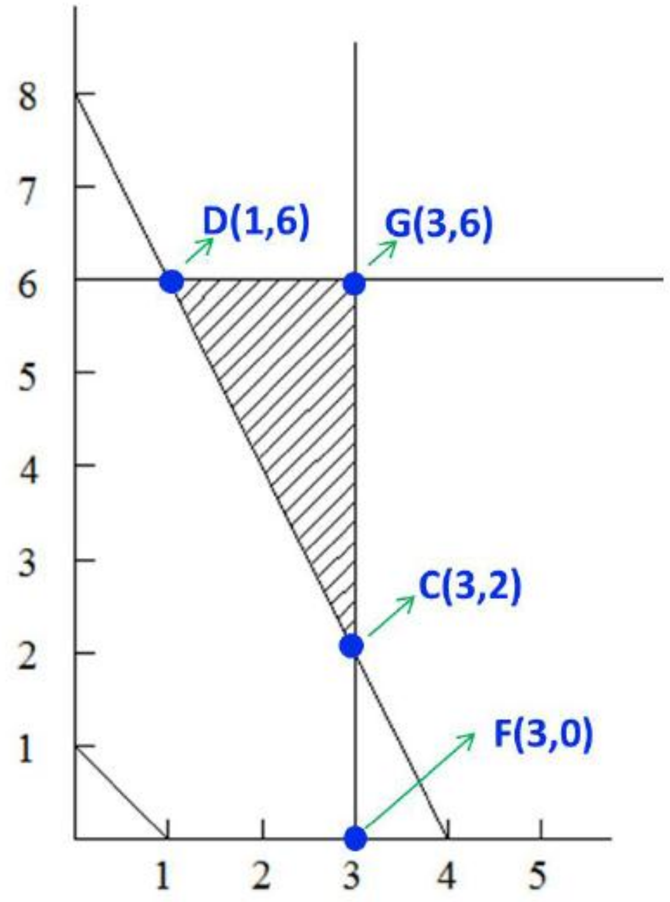
e) The objective function of Min $Z=x_1-2x_2$ is the same as the objective function of part (c) multiplied by -1. So the optimal point does not change, so E*(0,6) and Z*=-12.

f) If the second constraint is converted into equality, the feasible region will be a line segment CD and the optimal point is D*(2,6).



Exercise

g) When the second constraint is greater or equal, the feasible region changes as follows, the feasible region is CDG and the optimal point is G*(3,6) and Z*=45.



Exercises

Thanks

How to find us

www.optimizationcity.com

optimizationcity@gmail.com